Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Summer Term 2015
Problem Set 06
May 18, 2015

Complexity Theory

Due date: May 25, 2015 **before** *class!*

Problem 1 (10 Points)

Prove the existence of a nondeterministic universal Turing machine. That is, prove that there exists a representation scheme of NTMs, and an NTM NU such that for every string α, and input x, $NU(\alpha, x) = M_\alpha(x)$:

1. Prove that there exists a universal NTM NU such that if M_α halts on x within T steps, then NU halts on (α, x) within $cT \log T$ steps, where c is a constant only dependent on α.

2. Prove that there is such a universal NTM that runs on these inputs in at most cT steps.

Problem 2 (10 Points)

Show that $\mathbf{SPACE}(n) \neq \mathcal{NP}$. (Note that it is unknown if either class is contained in the other.)

Problem 3 (10 Points)

Define the class $\mathbf{E} = \bigcup_c \mathbf{DTIME}(2^{cn})$.

1. Is $\mathbf{E}$ closed under polynomial-time reductions?

2. Show that $\mathcal{P}^{\mathbf{E}} = \mathbf{EXP}$.

Problem 4 (10 Points)

1. Show that $\mathcal{NP}$ and co$\mathcal{NP}$ both are subsets of the set of languages which are polynomial-time Turing reducible to Sat.

2. Prove that if $\mathcal{NP}$ was equal to the set of languages which are polynomial-time Turing reducible to Sat, it would follow that $\mathcal{NP} =$ co$\mathcal{NP}$.