Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Winter Semester 2014/15
Problem Set 12
January 19th, 2015

Parallel Algorithms

Due date: January 26th, 2014 **before** *class!*

Problem 1 (10 Points)

Consider greedy x-y permutation routing on the $n \times n$ mesh, i.e., a packet with source (a, b) and destination (x, y) is first routed to its destined column (a, y), and then to its destined row (x, y). If the buffer size is more than 1, the farthest-packet-first strategy is used, i.e., of all the packets in the buffer, the one with the farthest remaining routing path is chosen to be sent next.

1. Show that the worst-case buffer size can be as high as $\frac{3}{2}n - \mathcal{O}(1)$ when a node can only forward one packet in every step.

2. Show that the worst-case buffer size can be as high as $\frac{2}{3}n - \mathcal{O}(1)$ when a node can forward any number of packets in every step (bounded by the number of incident edges of course).

Argue that these bounds are tight, i.e. there are no cases where the buffer size exceeds these bounds.

Problem 2 (10 Points)

A randomized oblivious routing scheme is α-*competitive* if, for every concurrent multicommodity flow problem P with given demands, it holds for the congestion C that $C \leq \alpha \cdot C_{opt}(P)$.

1. Consider an n-node ring network and prove that no oblivious routing scheme can have a competitive ratio of less than $2\left(1 - \frac{1}{n}\right)$.
   Hint: Consider the case where every node sends one unit of flow to its right neighbor and receives one unit from its left neighbor, and the case where only a single node sends one unit of flow to one of its neighbors.

2. Show that in an n-node ring network shortest-path-routing is 2-competitive.