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## Efficient Algorithms and Datastructures I

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### Question 1 (10 Points)

Solve the following recurrence relations using a generating function:

1.  $a_n = a_{n-1} + a_{n-2}$  for  $n \geq 2$  with  $a_0 = 0$  and  $a_1 = 1$ .
2.  $a_n = 5a_{n-1} - 8a_{n-2} + 4a_{n-3}$  for  $n \geq 3$  with  $a_0 = 1, a_1 = 3$  and  $a_2 = 11$ .

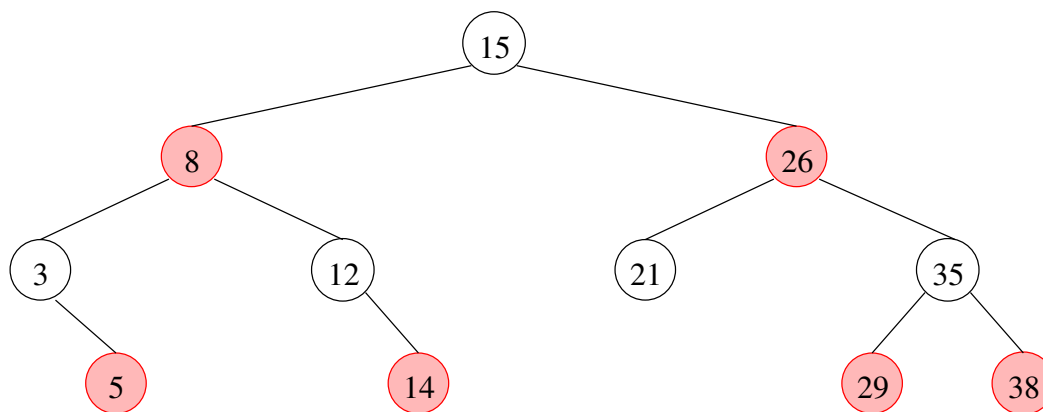
### Question 2 (10 Points)

Give tight asymptotic bounds for  $T(n)$ :

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

### Question 3 (10 Points)

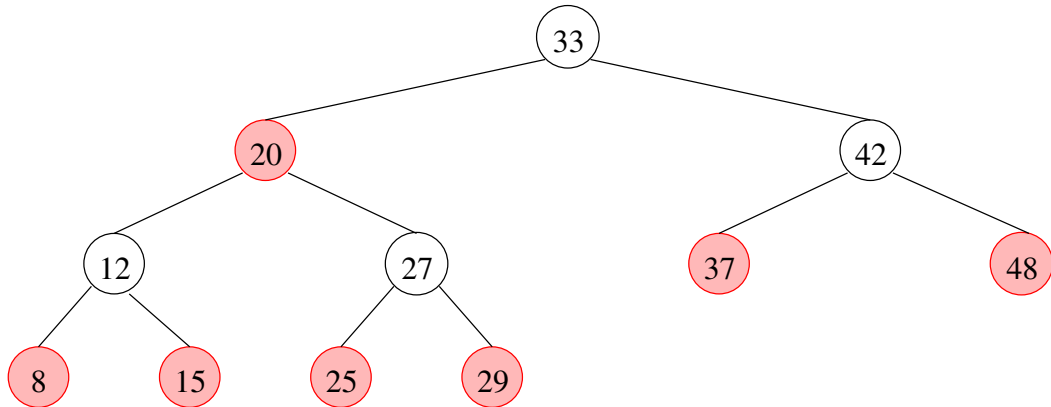
Carry out the following operations sequentially on the red-black tree shown below so that it remains a red-black tree and show what the tree looks like after each operation (always carry out each operation on the result of the previous operation):



1. Insert 10
2. Delete 29
3. Delete 21
4. Delete 3

### Question 4 (10 Points)

Carry out the following operations sequentially on the red-black tree shown below so that it remains a red-black tree and show what the tree looks like after each operation (always carry out each operation on the result of the previous operation):



1. Insert 5
2. Delete 27
3. Insert 27