Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
Prof. Dr. Ernst W. Mayr
Moritz Fuchs

Winter Semester 2014/15
Problem set 4
October 28, 2014

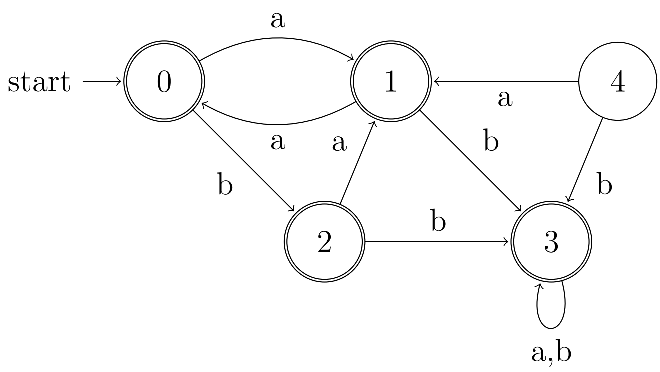
Automata and Formal Languages

Due November 04, 2014 before class!

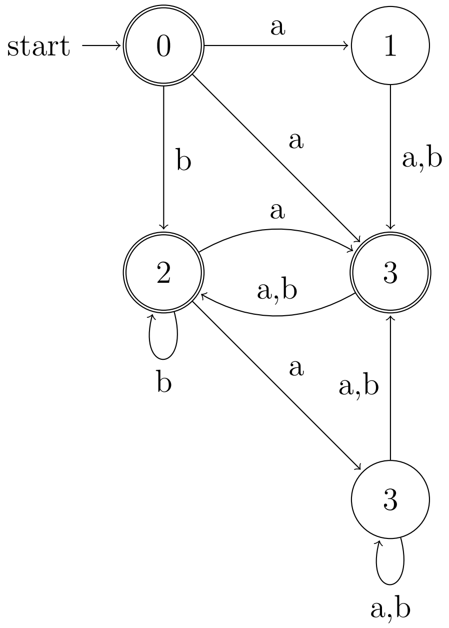
Exercise 1 (Universality - 10 points)

Decide if the following automata are universal or not. Use the methods from the lecture.

(a)



(b)



Exercise 2 (DFA Operations - 10 points)

(a) Construct DFAs M_1 and M_2 for

- $r_1 = aa^*bb$
- $r_2 = aabb^*$

(b) Construct automata recognizing

- $\mathcal{L}(M_1) \cup \mathcal{L}(M_2)$
- $\mathcal{L}(M_1) \cap \mathcal{L}(M_2)$
- $\mathcal{L}(M_1) \triangle \mathcal{L}(M_2)$

using the method from the lecture.

Exercise 3 (NFA Operations - 10 points)

(a) Construct NFAs N_1 and N_2 recognizing

- $r_1 = aaa^*aab$
- $r_2 = aa^*ab$

(b) Construct automata recognizing

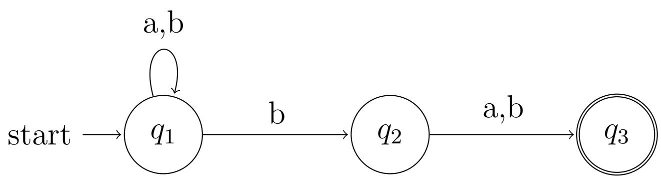
- $\mathcal{L}(N_1) \cup \mathcal{L}(N_2)$
- $\mathcal{L}(N_1) \cap \mathcal{L}(N_2)$

using the methods shown in the lecture.

Exercise 4 (Star-free expressions II - 10 points)

Recall the definition of star-free expressions from the previous problem set.
Let $A = (Q, \Sigma, \delta, q_{init}, F)$ be a finite automaton with states $Q = \{q_1, ..., q_n\}$. To every input letter $a \in \Sigma$ we associate a adjacency matrix $M_a \in \{0,1\}^{n \times n}$ in which the entry (i,j) is 1 if $\delta(q_i, a) = q_j$ and 0 else.

(a) Consider the following automaton:



The corresponding adjacency matrices are:

$$M_a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \; M_b = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

We define the map $h : \Sigma^* \to \{0,1\}^{3\times 3}$ as follows:

$$\begin{aligned} h(\epsilon) &= I_3 \\ h(a) &= M_a \\ h(b) &= M_b \\ h(a_1 a_2 ... a_k) &= h(a_1) \cdot h(a_2) \cdot \dots \cdot h(a_k) \end{aligned}$$

- For $w \in \Sigma^*$ describe what $h(w)$ represents.
- Draw a graph with vertex set $h(\Sigma^*) = \{h(w) \mid w \in \Sigma^*\}$ such that there is an edge between q and q' labeled by a if $q \cdot h(a) = q'$.
  Start drawing the graph at $h(\epsilon)$.
- How can you obtain a deterministic automaton accepting the same language as the original non-deterministic automaton?

(b) Let L be a regular language and M the minimal DFA accepting L with states $Q = \{q_1, ..., q_n\}$. We define h similarly as in the previous exercise and denote by S_L the set $h(\Sigma^*)$. S_L is called the *syntactic monoid* of L.
A theorem by Schützenberger[1] states that L is representable by a star-free expression iff $M^n = M^{n+1}$ for all $M \in S_L$ with $n = |Q|$.

- Show that $\mathcal{L}((aa)^*)$ is not representable by a star-free expression.
- Show that $\mathcal{L}((ab + ba)^*)$ is representable by a star-free expression.

Literatur

[1] Marcel Paul Schützenberger. On finite monoids having only trivial subgroups. *Information and control*, 8(2):190–194, 1965.