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## Parallel Algorithms

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*Due Date: January 29, 2013 before class!*

### **Problem 1 (10 Points)**

Given two families of graphs  $\mathcal{G}$  and  $\mathcal{H}$  and an embedding of any graph  $G \in \mathcal{G}$  into some graph of  $\mathcal{H}$  with constant expansion, load, congestion, and dilation, show that any graph  $G \in \mathcal{G}$  can be simulated by some graph in  $\mathcal{H}$  with constant slowdown and constant loss in efficiency.

### **Problem 2 (10 Points)**

Show that the bounds in the Theorem about the flip-bit algorithm (Theorem 3.4. in Leighton's book) hold with probability  $1 - N^{-\alpha}$  for any constant  $\alpha$ .

### **Problem 3 (10 Points)**

Show that the number of hypercube nodes with  $i$  1s is at most  $O(N/\sqrt{\log N})$  for any  $i$ .

### **Problem 4 (10 Points)**

Given any deterministic algorithm for dynamically embedding an  $N$ -node binary tree in an  $N$ -node hypercube with load  $L$ , show that there is a way to grow a binary tree that will force the dilation of the embedding to be  $\Omega(\sqrt{\log N}/L^2)$ .