Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Winter Semester 2012/13
Problem Set 6
November 27, 2012

Parallel Algorithms

Due Date: December 4, 2012 **before** *class!*

Problem 1 (10 Points)

Show that the maximum queue size is at most $\frac{2}{3}\sqrt{N}$ for the basic greedy algorithm for the routing problem.

Problem 2 (10 Points)

Extend the Lemma about the Chernov bound to show that

$$\text{Prob}(X \leq \gamma P) \leq e^{\left(P-\gamma P-(n-\gamma P)\ln\left(\frac{n-\gamma P}{n-P}\right)\right)\frac{P}{n-P}}$$

for $\gamma < 1$.
Hint: Reverse the roles of $X_i = 0$ and $X_i = 1$.

Problem 3 (10 Points)

Show that with probability close to 1, the basic greedy algorithm solves a random routing problem in $2\sqrt{N} - \Omega(N^{1/4})$ steps.
Hint: You will have to show that, with probability close to 1, no packet will have to travel more than $2\sqrt{N} - \Omega(N^{1/4})$ distance.

Problem 4 (10 Points)

Show Hall's marriage theorem:
Let $G = (U, V, E)$ be a bipartite graph with $|U| = n$ and $|V| = m$ and $m \leq n$. Then there exists a matching of cardinality m if and only if for every subset $V' \subseteq V$ it holds that $|V'| \leq |N(V)|$, where $N(V)$ denotes the set of neighboring vertices to vertices in V.