Technische Universität München
Fakultät für Informatik
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Spring Semester 2012
Problem Set 2
May 8, 2012

Complexity Theory

Due date: May 15, 2012 **before** *class!*

Problem 1 (10 Points)

(i) One can easily show that the polynomial-time many-to-one reduction $\preceq_m^p$ is reflexive (i.e. $A \preceq_m^p A$ for all languages A) and transitive (i.e., if $A \preceq_m^p B$ and $B \preceq_m^p C$, then $A \preceq_m^p C$). But is it also commutative (i.e., if $A \preceq_m^p B$, then $B \preceq_m^p A$)?

(ii) Show or disprove: $\mathcal{NP}$ is closed under union or intersection, respectively. (Meaning that if $L_1, L_2 \in \mathcal{NP}$, then $L_1 \cup L_2 \in \mathcal{NP}$ or $L_1 \cap L_2 \in \mathcal{NP}$, respectively.)

Problem 2 (10 Points)

(Berman 1978) A *unary* language contains strings of the form 1^m, i.e. strings of m ones for some $m > 0$. Show that if a $\mathcal{NP}$-complete unary language exists, then $\mathcal{P} = \mathcal{NP}$.

Problem 3 (10 Points)

Prove that $\mathcal{P} \subseteq \mathcal{NP} \cap \text{co-}\mathcal{NP}$.

Problem 4 (10 Points)

Let Tautology $= \{\varphi : \varphi \text{ is a boolean formula that is satisfied by every assignment}\}$. Show that $\mathcal{NP} = \text{co-}\mathcal{NP}$ if and only if 3Sat and Tautology are polynomial-time reducible to one another.