Supersymmetry and Quantum Hall effect in graphene

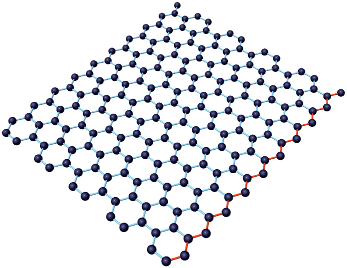
Ervand Kandelaki

Lehrstuhl für Theoretische Festköperphysik
Institut für Theoretische Physik IV
Universität Erlangen-Nürnberg

March 14, 2007

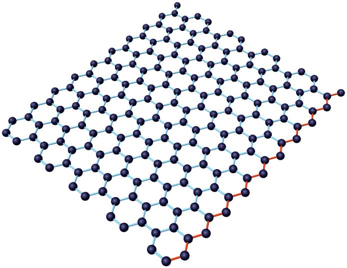
Graphene as a new material

- Graphene is a single planar sheet of carbon atoms



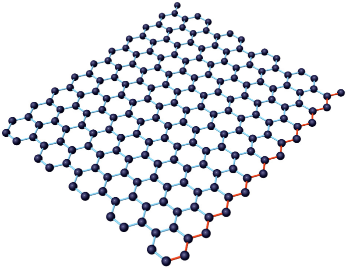
Graphene as a new material

- Graphene is a single planar sheet of carbon atoms
- Can be regarded as a layer of graphite



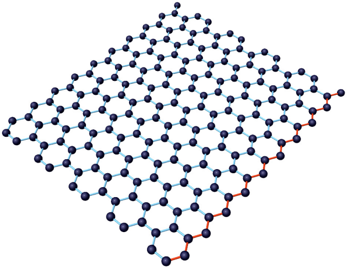
Graphene as a new material

- Graphene is a single planar sheet of carbon atoms
- Can be regarded as a layer of graphite
- Many interesting properties (recall talk by Anton Lopatin)



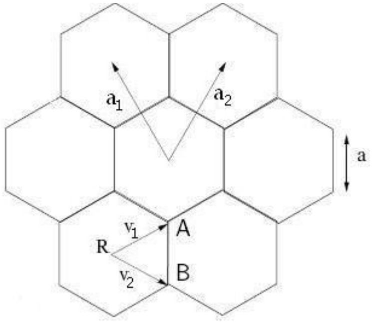
Graphene as a new material

- Graphene is a single planar sheet of carbon atoms
- Can be regarded as a layer of graphite
- Many interesting properties (recall talk by Anton Lopatin)
- Especially: specific Hamiltonian & unusual Hall effect



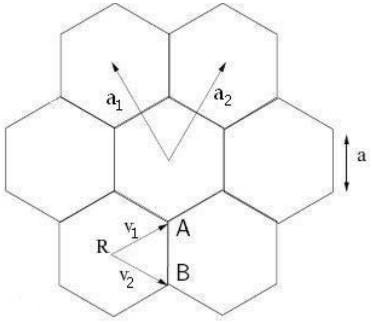
Graphene Lattice

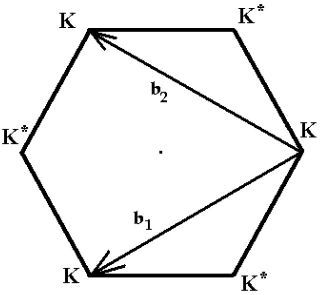
- Hexagonal 2D-lattice with diatomic basis



Graphene Lattice

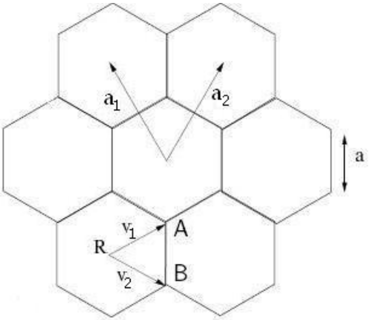
- Hexagonal 2D-lattice with diatomic basis
- Both in real and reciprocal space

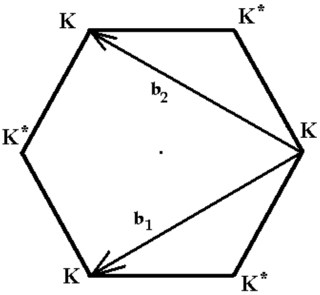




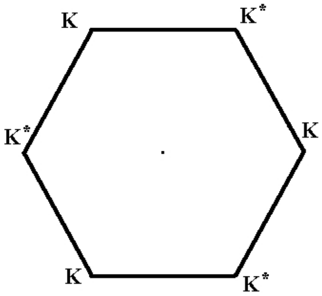
Graphene Lattice

- Hexagonal 2D-lattice with diatomic basis
- Both in real and reciprocal space
- $\vec{R}$ = lattice vector $\Rightarrow \vec{v}_1 + \vec{R} \equiv A$, $\vec{v}_2 + \vec{R} \equiv B$



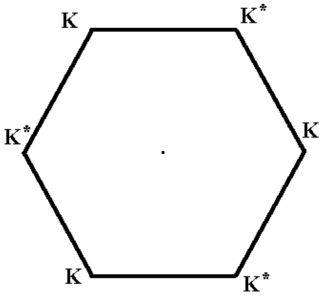


Goal



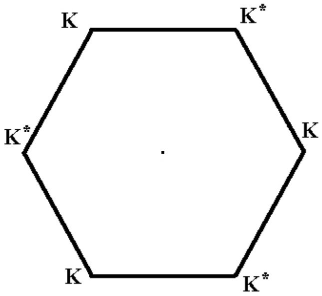
- The Brillouin zone contains two independent points: K and K*

Goal



- The Brillouin zone contains two independent points: K and K*
- They are symmetric to each other with respect to time-inversion $t \to -t$

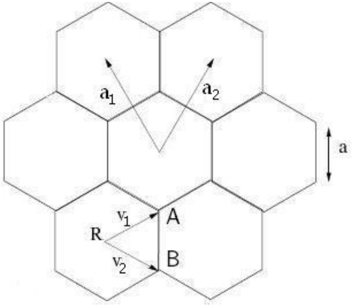
Goal



- The Brillouin zone contains two independent points: K and K*
- They are symmetric to each other with respect to time-inversion $t \longrightarrow -t$
- The goal is the effective Hamiltonian around K and K*

Tight Binding

$$\psi_{\vec{k}}(\vec{r}) = \sum_{\vec{R}} e^{i\vec{k}\vec{R}} \chi(\vec{r} - \vec{R})$$

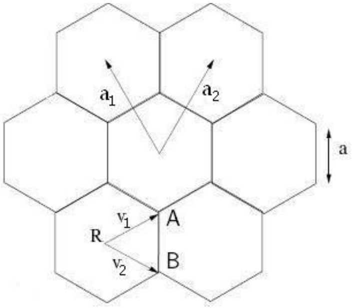


Tight Binding

$$\psi_{\vec{k}}(\vec{r}) = \sum_{\vec{R}} e^{i\vec{k}\vec{R}} \chi(\vec{r} - \vec{R})$$

$$\psi_{\vec{k}}(\vec{r}) = \alpha \sum_{\vec{R}} e^{i\vec{k}(\vec{R}+\vec{v}_1)} f(\vec{r} - \vec{R} - \vec{v}_1) +$$

$$+ \beta \sum_{\vec{R}} e^{i\vec{k}(\vec{R}+\vec{v}_2)} f(\vec{r} - \vec{R} - \vec{v}_2)$$

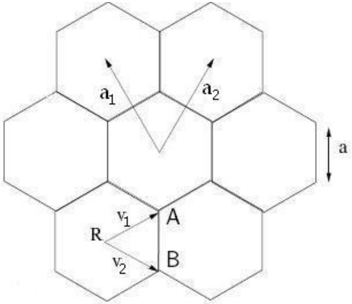


Tight Binding

$$\psi_{\vec{k}}(\vec{r}) = \sum_{\vec{R}} e^{i\vec{k}\vec{R}} \chi(\vec{r} - \vec{R})$$

$$\psi_{\vec{k}}(\vec{r}) = \alpha \sum_{\vec{R}} e^{i\vec{k}(\vec{R}+\vec{v}_1)} f(\vec{r} - \vec{R} - \vec{v}_1) + $$

$$+ \beta \sum_{\vec{R}} e^{i\vec{k}(\vec{R}+\vec{v}_2)} f(\vec{r} - \vec{R} - \vec{v}_2)$$

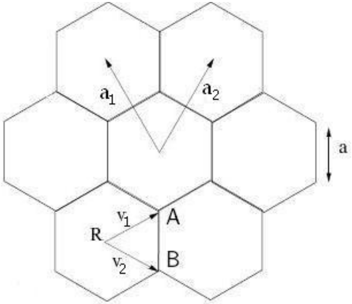


$$= \sum_{\vec{R}} e^{i\vec{k}\vec{R}} \left[\alpha e^{i\vec{k}\vec{v}_1} f(\vec{r} - \vec{R} - \vec{v}_1) + \beta e^{i\vec{k}\vec{v}_2} f(\vec{r} - \vec{R} - \vec{v}_2) \right]$$

Tight Binding

$$\psi_{\vec{k}}(\vec{r}) = \sum_{\vec{R}} e^{i\vec{k}\vec{R}} \chi(\vec{r} - \vec{R})$$

$$\psi_{\vec{k}}(\vec{r}) = \alpha \sum_{\vec{R}} e^{i\vec{k}(\vec{R}+\vec{v}_1)} f(\vec{r} - \vec{R} - \vec{v}_1) + \\ + \beta \sum_{\vec{R}} e^{i\vec{k}(\vec{R}+\vec{v}_2)} f(\vec{r} - \vec{R} - \vec{v}_2)$$



$$= \sum_{\vec{R}} e^{i\vec{k}\vec{R}} \left[\alpha e^{i\vec{k}\vec{v}_1} f(\vec{r} - \vec{R} - \vec{v}_1) + \beta e^{i\vec{k}\vec{v}_2} f(\vec{r} - \vec{R} - \vec{v}_2) \right]$$

$$=: \sum_{\vec{R}} e^{i\vec{k}\vec{R}} \left(\alpha \varphi^{\alpha}_{\vec{k}}(\vec{r}) + \beta \varphi^{\beta}_{\vec{k}}(\vec{r}) \right) = \alpha \psi^{\alpha}_{\vec{k}}(\vec{r}) + \beta \psi^{\beta}_{\vec{k}}(\vec{r})$$

Obtaining Hamiltonian

$$\hat{H}\psi_{\vec{k}} = \varepsilon_{\vec{k}}\psi_{\vec{k}}, \quad \psi_{\vec{k}} = \alpha\psi^{\alpha}_{\vec{k}} + \beta\psi^{\beta}_{\vec{k}}$$

Obtaining Hamiltonian

$$\hat{H}\psi_{\vec{k}} = \varepsilon_{\vec{k}}\psi_{\vec{k}}, \quad \psi_{\vec{k}} = \alpha\psi_{\vec{k}}^{\alpha} + \beta\psi_{\vec{k}}^{\beta}$$

$\hat{H}$ acts on vectors in 2-dimensional space spanned by $\left(\begin{smallmatrix}1\\0\end{smallmatrix}\right) \equiv \psi_{\vec{k}}^{\alpha} \equiv A-$sublattice and $\left(\begin{smallmatrix}0\\1\end{smallmatrix}\right) \equiv \psi_{\vec{k}}^{\beta} \equiv B-$sublattice

Obtaining Hamiltonian

$$\hat{H}\psi_{\vec{k}} = \varepsilon_{\vec{k}}\psi_{\vec{k}}, \quad \psi_{\vec{k}} = \alpha\psi_{\vec{k}}^{\alpha} + \beta\psi_{\vec{k}}^{\beta}$$

$\hat{H}$ acts on vectors in 2-dimensional space spanned by $\left(\begin{smallmatrix}1\\0\end{smallmatrix}\right) \equiv \psi_{\vec{k}}^{\alpha} \equiv\ A$−sublattice and $\left(\begin{smallmatrix}0\\1\end{smallmatrix}\right) \equiv \psi_{\vec{k}}^{\beta} \equiv\ B$−sublattice

$$\hat{H}_{\alpha\beta} = \left\langle \psi_{\vec{k}}^{\alpha} \right| \hat{H} \left| \psi_{\vec{k}}^{\beta} \right\rangle =$$

Obtaining Hamiltonian

$$\hat{H}\psi_{\vec{k}} = \varepsilon_{\vec{k}}\psi_{\vec{k}}, \quad \psi_{\vec{k}} = \alpha\psi_{\vec{k}}^{\alpha} + \beta\psi_{\vec{k}}^{\beta}$$

$\hat{H}$ acts on vectors in 2-dimensional space spanned by $\left(\begin{smallmatrix}1\\0\end{smallmatrix}\right) \equiv \psi_{\vec{k}}^{\alpha} \equiv$ $A-$sublattice and $\left(\begin{smallmatrix}0\\1\end{smallmatrix}\right) \equiv \psi_{\vec{k}}^{\beta} \equiv$ $B-$sublattice

$$\hat{H}_{\alpha\beta} = \left\langle \psi_{\vec{k}}^{\alpha} \right| \hat{H} \left| \psi_{\vec{k}}^{\beta} \right\rangle =$$

$$= \sum_{\vec{R},\vec{R}'} e^{-i\vec{k}(\vec{R}+\vec{v}_1-\vec{R}'-\vec{v}_2)} \left\langle f(\vec{r}-\vec{R}-\vec{v}_1) \right| \hat{H} \left| f(\vec{r}-\vec{R}'-\vec{v}_2) \right\rangle$$

Obtaining Hamiltonian

$$\hat{H}\psi_{\vec{k}} = \varepsilon_{\vec{k}}\psi_{\vec{k}}, \quad \psi_{\vec{k}} = \alpha\psi_{\vec{k}}^{\alpha} + \beta\psi_{\vec{k}}^{\beta}$$

$\hat{H}$ acts on vectors in 2-dimensional space spanned by $\left(\begin{smallmatrix}1\\0\end{smallmatrix}\right) \equiv \psi_{\vec{k}}^{\alpha} \equiv A-$sublattice and $\left(\begin{smallmatrix}0\\1\end{smallmatrix}\right) \equiv \psi_{\vec{k}}^{\beta} \equiv B-$sublattice

$$\hat{H}_{\alpha\beta} = \left\langle \psi_{\vec{k}}^{\alpha} \right| \hat{H} \left| \psi_{\vec{k}}^{\beta} \right\rangle =$$

$$= \sum_{\vec{R},\vec{R}'} e^{-i\vec{k}(\vec{R}+\vec{v}_1-\vec{R}'-\vec{v}_2)} \left\langle f(\vec{r}-\vec{R}-\vec{v}_1) \right| \hat{H} \left| f(\vec{r}-\vec{R}'-\vec{v}_2) \right\rangle$$

$$= \sum_{\vec{R},\vec{R}'} e^{-i\vec{k}(\vec{R}+\vec{v}_1-\vec{R}'-\vec{v}_2)} \left\langle f(\vec{r}-(\vec{R}+\vec{v}_1-\vec{R}'-\vec{v}_2)) \right| \hat{H} \left| f(\vec{r}) \right\rangle \approx$$

Obtaining Hamiltonian

$$\hat{H}\psi_{\vec{k}} = \varepsilon_{\vec{k}}\psi_{\vec{k}}, \quad \psi_{\vec{k}} = \alpha\psi^{\alpha}_{\vec{k}} + \beta\psi^{\beta}_{\vec{k}}$$

$\hat{H}$ acts on vectors in 2-dimensional space spanned by $\left(\begin{smallmatrix}1\\0\end{smallmatrix}\right) \equiv \psi^{\alpha}_{\vec{k}} \equiv A-$sublattice and $\left(\begin{smallmatrix}0\\1\end{smallmatrix}\right) \equiv \psi^{\beta}_{\vec{k}} \equiv B-$sublattice

$$\hat{H}_{\alpha\beta} = \left\langle \psi^{\alpha}_{\vec{k}} \middle| \hat{H} \middle| \psi^{\beta}_{\vec{k}} \right\rangle =$$

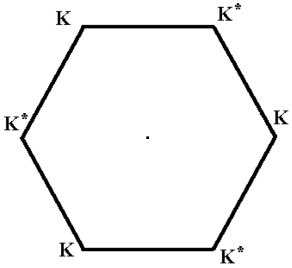
$$= \sum_{\vec{R},\vec{R}'} e^{-i\vec{k}(\vec{R}+\vec{v}_1-\vec{R}'-\vec{v}_2)} \left\langle f(\vec{r}-\vec{R}-\vec{v}_1) \middle| \hat{H} \middle| f(\vec{r}-\vec{R}'-\vec{v}_2) \right\rangle$$

$$= \sum_{\vec{R},\vec{R}'} e^{-i\vec{k}(\vec{R}+\vec{v}_1-\vec{R}'-\vec{v}_2)} \left\langle f(\vec{r}-(\vec{R}+\vec{v}_1-\vec{R}'-\vec{v}_2)) \middle| \hat{H} \middle| f(\vec{r}) \right\rangle \approx$$

$$\approx \sum_{j} t e^{-i\vec{k}\vec{u}_j}, \quad \vec{u}_j = \vec{R}+\vec{v}_1-\vec{R}'-\vec{v}_2, \; |\vec{u}_j| = |\vec{v}_1-\vec{v}_2|$$

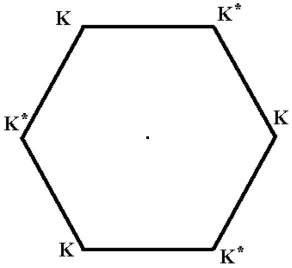
Obtaining Hamiltonian (2)

$$\hat{H} = \begin{pmatrix} 0 & t\sum_j e^{-i\vec{k}\vec{u}_j} \\ t\sum_j e^{i\vec{k}\vec{u}_j} & 0 \end{pmatrix}$$



Obtaining Hamiltonian (2)

$$\hat{H} = \begin{pmatrix} 0 & t\sum_j e^{-i\vec{k}\vec{u}_j} \\ t\sum_j e^{i\vec{k}\vec{u}_j} & 0 \end{pmatrix}$$

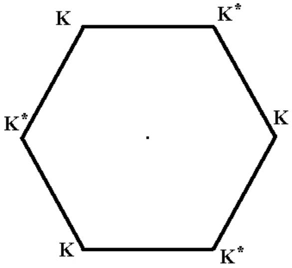


Taylor-expansion around $\vec{k}_K$ $\left(\vec{k} = \vec{k}_K + \vec{k'}\right)$ yields

$$\hat{H} = \begin{pmatrix} 0 & (k'_x - ik'_y)v \\ (k'_x + ik'_y)v & 0 \end{pmatrix}$$

Obtaining Hamiltonian (2)

$$\hat{H} = \begin{pmatrix} 0 & t\sum_j e^{-i\vec{k}\vec{u}_j} \\ t\sum_j e^{i\vec{k}\vec{u}_j} & 0 \end{pmatrix}$$



Taylor-expansion around $\vec{k}_K$ $\left(\vec{k} = \vec{k}_K + \vec{k'}\right)$ yields

$$\hat{H} = \begin{pmatrix} 0 & (k'_x - ik'_y)v \\ (k'_x + ik'_y)v & 0 \end{pmatrix}$$

More general, using effective mass method:

$$\hat{H} = \begin{pmatrix} 0 & (\hat{p}_x - i\hat{p}_y)v \\ (\hat{p}_x + i\hat{p}_y)v & 0 \end{pmatrix}$$

Dirac-Hamiltonian

$$\hat{H} = \begin{pmatrix} 0 & (\hat{p}_x - i\hat{p}_y)v \\ (\hat{p}_x + i\hat{p}_y)v & 0 \end{pmatrix}$$

- This is the Dirac Hamiltonian of a relativistic particle!

Dirac-Hamiltonian

$$\hat{H} = \begin{pmatrix} 0 & (\hat{p}_x - i\hat{p}_y)v \\ (\hat{p}_x + i\hat{p}_y)v & 0 \end{pmatrix}$$

- This is the Dirac Hamiltonian of a relativistic particle!
- Only important difference: no spin involved

Dirac-Hamiltonian

$$\hat{H} = \begin{pmatrix} 0 & (\hat{p}_x - i\hat{p}_y)v \\ (\hat{p}_x + i\hat{p}_y)v & 0 \end{pmatrix}$$

- This is the Dirac Hamiltonian of a relativistic particle!
- Only important difference: no spin involved
- Extremely relativistic (no mc^2-term)

Dirac-Hamiltonian

$$\hat{H} = \begin{pmatrix} 0 & (\hat{p}_x - i\hat{p}_y)v \\ (\hat{p}_x + i\hat{p}_y)v & 0 \end{pmatrix}$$

- This is the Dirac Hamiltonian of a relativistic particle!
- Only important difference: no spin involved
- Extremely relativistic (no mc^2-term)
- Also called Weyl-Hamiltonian

Dirac-Hamiltonian

$$\hat{H} = \begin{pmatrix} 0 & (\hat{p}_x - i\hat{p}_y)v \\ (\hat{p}_x + i\hat{p}_y)v & 0 \end{pmatrix}$$

- This is the Dirac Hamiltonian of a relativistic particle!
- Only important difference: no spin involved
- Extremely relativistic (no mc^2-term)
- Also called Weyl-Hamiltonian
- Describes particles and antiparticles, e.g. neutrinos

Dirac-Hamiltonian

$$\hat{H} = \begin{pmatrix} 0 & (\hat{p}_x - i\hat{p}_y)v \\ (\hat{p}_x + i\hat{p}_y)v & 0 \end{pmatrix}$$

- This is the Dirac Hamiltonian of a relativistic particle!
- Only important difference: no spin involved
- Extremely relativistic (no mc^2-term)
- Also called Weyl-Hamiltonian
- Describes particles and antiparticles, e.g. neutrinos
- This Hamiltonian acts around K, a similar one around K*

Eigenvectors without magnetic field

$$\hat{H} = \begin{pmatrix} 0 & (\hat{p}_x - i\hat{p}_y)v \\ (\hat{p}_x + i\hat{p}_y)v & 0 \end{pmatrix}$$

Eigenvectors without magnetic field

$$\hat{H} = \begin{pmatrix} 0 & (\hat{p}_x - i\hat{p}_y)v \\ (\hat{p}_x + i\hat{p}_y)v & 0 \end{pmatrix}$$

Eigenvectors

$$\psi_{\vec{k}} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} e^{i\vec{k}\vec{r}}, \ \vec{p} = \hbar\vec{k}$$

Eigenvectors without magnetic field

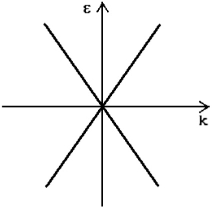
$$\hat{H} = \begin{pmatrix} 0 & (\hat{p}_x - i\hat{p}_y)v \\ (\hat{p}_x + i\hat{p}_y)v & 0 \end{pmatrix}$$

Eigenvectors

$$\psi_{\vec{k}} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} e^{i\vec{k}\vec{r}}, \ \vec{p} = \hbar\vec{k}$$

Using of Schroedinger-equation leads to

$$\psi_{\vec{k}}^{\pm} = \frac{1}{\sqrt{2}} e^{i\vec{k}\vec{r}} \begin{pmatrix} \pm i e^{-i\frac{\theta_k}{2}} \\ e^{i\frac{\theta_k}{2}} \end{pmatrix}, \ \varepsilon_{\pm} = \pm\hbar v \left|\vec{k}\right|,$$



where $k_x + ik_y = \left|\vec{k}\right| e^{i\left(\frac{\pi}{2} - \theta_k\right)}$

Introducing Pauli-matrices

No "normal" spin appears in this probem, but we use quasi-spin.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$$

Introducing Pauli-matrices

No "normal" spin appears in this probem, but we use quasi-spin.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \sigma_\pm = \frac{1}{2}\left(\sigma_x \pm i\sigma_y\right)$$

Introducing Pauli-matrices

No "normal" spin appears in this probem, but we use quasi-spin.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$$

$$\Rightarrow \hat{H} = \begin{pmatrix} 0 & \hbar(k_x - ik_y)v \\ \hbar(k_x + ik_y)v & 0 \end{pmatrix} = \hbar v \sigma_x k_x + \hbar v \sigma_y k_y =$$

Introducing Pauli-matrices

No "normal" spin appears in this probem, but we use quasi-spin.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$$

$$\Rightarrow \hat{H} = \begin{pmatrix} 0 & \hbar(k_x - ik_y)v \\ \hbar(k_x + ik_y)v & 0 \end{pmatrix} = \hbar v \sigma_x k_x + \hbar v \sigma_y k_y =$$

$$= \hbar v \vec{\sigma}_\perp \vec{k}_\perp = \hbar v \left|\vec{k}_\perp\right| \vec{\sigma}_\perp \frac{\vec{k}_\perp}{\left|\vec{k}_\perp\right|} =: \hbar v \left|\vec{k}_\perp\right| \chi_{\vec{k}}$$

$$\vec{\sigma}_\perp = (\sigma_x, \sigma_y),\ \vec{k}_\perp = (k_x, k_y),\ k_\pm = k_x \pm ik_y$$

Chirality operator

$$\hat{H} = \hbar v \left|\vec{k}_\perp\right| \chi_{\vec{k}}$$

- $\chi_{\vec{k}} = \vec{\sigma}_\perp \frac{\vec{k}_\perp}{\left|\vec{k}_\perp\right|}$ chirality operator

Chirality operator

$$\hat{H} = \hbar v \left|\vec{k}_\perp\right| \chi_{\vec{k}}$$

- $\chi_{\vec{k}} = \vec{\sigma}_\perp \frac{\vec{k}_\perp}{\left|\vec{k}_\perp\right|}$ chirality operator
- Commutes with Hamiltonian $\Rightarrow$ conservation of chirality

Chirality operator

$$\hat{H} = \hbar v \left| \vec{k}_\perp \right| \chi_{\vec{k}}$$

- $\chi_{\vec{k}} = \vec{\sigma}_\perp \frac{\vec{k}_\perp}{\left|\vec{k}_\perp\right|}$ chirality operator
- Commutes with Hamiltonian $\Rightarrow$ conservation of chirality
- $\Rightarrow$ chirality is a good quantum number, quasi-spin is a bad one

Chirality operator

$$\hat{H} = \hbar v \left| \vec{k}_\perp \right| \chi_{\vec{k}}$$

- $\chi_{\vec{k}} = \vec{\sigma}_\perp \frac{\vec{k}_\perp}{\left|\vec{k}_\perp\right|}$ chirality operator
- Commutes with Hamiltonian $\Rightarrow$ conservation of chirality
- $\Rightarrow$ chirality is a good quantum number, quasi-spin is a bad one
- Solution can be described completely by $\vec{k}$ and $\chi_{\vec{k}}$

Hamiltonian in magnetic field

$$\vec{A} = (0, Bx)\,,\ \ \vec{B} = \nabla \times \vec{A} \quad \text{(Landau gauge)}$$

Hamiltonian in magnetic field

$$\vec{A} = (0, Bx)\,,\ \vec{B} = \nabla \times \vec{A} \quad \text{(Landau gauge)}$$

Introducing magnetic field by replacements

$$\hat{\vec{p}} = -i\hbar\nabla \rightarrow \hat{\vec{\pi}} = \hat{\vec{p}} - \frac{e}{c}\vec{A}, \quad \hat{H} = v\vec{\sigma}\vec{p} \rightarrow v\vec{\sigma}\vec{\pi} = v\vec{\sigma}_{\perp}\hat{\vec{\pi}}_{\perp}$$

Hamiltonian in magnetic field

$$\vec{A} = (0, Bx)\,, \;\; \vec{B} = \nabla \times \vec{A} \quad \text{(Landau gauge)}$$

Introducing magnetic field by replacements

$$\hat{\vec{p}} = -i\hbar\nabla \rightarrow \hat{\vec{\pi}} = \hat{\vec{p}} - \frac{e}{c}\vec{A}, \quad \hat{H} = v\vec{\sigma}\vec{p} \rightarrow v\vec{\sigma}\vec{\pi} = v\vec{\sigma}_\perp\hat{\vec{\pi}}_\perp$$

Introduce $\hat{\vec{\pi}}_\perp = (\hat{\pi}_x, \hat{\pi}_y)$

$$\vec{\sigma}_\perp\vec{p}_\perp = \sigma_+ p_- + \sigma_- p_+ \;\Rightarrow\; \vec{\sigma}_\perp\hat{\vec{\pi}}_\perp = \sigma_+\hat{\pi}_- + \sigma_-\hat{\pi}_+$$

Hamiltonian in magnetic field

$$\vec{A} = (0, Bx)\,,\ \ \vec{B} = \nabla \times \vec{A} \quad \text{(Landau gauge)}$$

Introducing magnetic field by replacements

$$\hat{\vec{p}} = -i\hbar\nabla \to \hat{\vec{\pi}} = \hat{\vec{p}} - \frac{e}{c}\vec{A}, \quad \hat{H} = v\vec{\sigma}\vec{p} \to v\vec{\sigma}\vec{\pi} = v\vec{\sigma}_\perp\hat{\vec{\pi}}_\perp$$

Introduce $\hat{\vec{\pi}}_\perp = (\hat{\pi}_x, \hat{\pi}_y)$

$$\vec{\sigma}_\perp\vec{p}_\perp = \sigma_+ p_- + \sigma_- p_+ \ \Rightarrow\ \vec{\sigma}_\perp\hat{\vec{\pi}}_\perp = \sigma_+\hat{\pi}_- + \sigma_-\hat{\pi}_+$$

$$\hat{\pi}_\pm = \hat{p}_\pm \mp i\frac{e}{c}Bx \ \Rightarrow [\hat{\pi}_+, \hat{\pi}_-] = 2\frac{e\hbar}{c}B$$

Analogy with ladder operators

$$[\hat{\pi}_+, \hat{\pi}_-] = 2\frac{e\hbar}{c}B = 2\hbar^2\frac{|eB|}{c\hbar}\mathsf{sgn}(eB) = 2\frac{\hbar^2}{l_H^2}\mathsf{sgn}(eB) =: \gamma^2\mathsf{sgn}(eB)$$

Analogy with ladder operators

$$[\hat{\pi}_+, \hat{\pi}_-] = 2\frac{e\hbar}{c}B = 2\hbar^2\frac{|eB|}{c\hbar}\mathit{sgn}(eB) = 2\frac{\hbar^2}{l_H^2}\mathit{sgn}(eB) =: \gamma^2\mathit{sgn}(eB)$$

$$l_H^2 = \frac{c\hbar}{|eB|} \rightarrow \text{ Magnetic length}$$

Analogy with ladder operators

$$[\hat{\pi}_+, \hat{\pi}_-] = 2\frac{e\hbar}{c}B = 2\hbar^2\frac{|eB|}{c\hbar}sgn(eB) = 2\frac{\hbar^2}{l_H^2}sgn(eB) =: \gamma^2 sgn(eB)$$

$$l_H^2 = \frac{c\hbar}{|eB|} \rightarrow \text{ Magnetic length}$$

Recall: raising and lowering operators of HO are defined as

$$a\left|n\right\rangle = \sqrt{n}\left|n-1\right\rangle, \; a^\dagger\left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle$$

Analogy with ladder operators

$$[\hat{\pi}_+, \hat{\pi}_-] = 2\frac{e\hbar}{c}B = 2\hbar^2\frac{|eB|}{c\hbar}sgn(eB) = 2\frac{\hbar^2}{l_H^2}sgn(eB) =: \gamma^2 sgn(eB)$$

$$l_H^2 = \frac{c\hbar}{|eB|} \rightarrow \text{ Magnetic length}$$

Recall: raising and lowering operators of HO are defined as

$$a\left|n\right\rangle = \sqrt{n}\left|n-1\right\rangle, \;\; a^\dagger\left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle$$

$$\left[a, a^\dagger\right] = 1, \;\; a^\dagger a = \hat{N}$$

Analogy with ladder operators

$$[\hat{\pi}_+, \hat{\pi}_-] = 2\frac{e\hbar}{c}B = 2\hbar^2\frac{|eB|}{c\hbar}\mathit{sgn}(eB) = 2\frac{\hbar^2}{l_H^2}\mathit{sgn}(eB) =: \gamma^2\mathit{sgn}(eB)$$

$$l_H^2 = \frac{c\hbar}{|eB|} \rightarrow \text{ Magnetic length}$$

Recall: raising and lowering operators of HO are defined as

$$a\left|n\right\rangle = \sqrt{n}\left|n-1\right\rangle, \;\; a^\dagger\left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle$$

$$\left[a, a^\dagger\right] = 1, \;\; a^\dagger a = \hat{N}$$

$\hat{\vec{\pi}}_\pm$ can therefore be understood as raising/lowering operators depending on $\mathit{sgn}(eB)$

What is supersymmetry?

- We found the analogy with ladder operators of the harmonic oscillator

What is supersymmetry?

- We found the analogy with ladder operators of the harmonic oscillator
- More general: supesymmetic operators

What is supersymmetry?

- We found the analogy with ladder operators of the harmonic oscillator
- More general: supesymmetic operators
- We consider a system with bosons and fermions

What is supersymmetry?

- We found the analogy with ladder operators of the harmonic oscillator
- More general: supesymmetic operators
- We consider a system with bosons and fermions
- Supersymmetry = symmetry bosons $\longleftrightarrow$ fermions

What is supersymmetry?

- We found the analogy with ladder operators of the harmonic oscillator
- More general: supesymmetic operators
- We consider a system with bosons and fermions
- Supersymmetry $=$ symmetry bosons $\longleftrightarrow$ fermions
- States are $|n_B, n_F\rangle$, $n_B = 0, 1, 2, \ldots,\ n_F = 0, 1$

Suppersymmetric operators

Annihilation and creation operators:

Suppersymmetric operators

Annihilation and creation operators:

$$b\left|n_B, n_F\right\rangle = \sqrt{n_B}\left|n_B - 1, n_F\right\rangle, \; b^{\dagger}\left|n_B, n_F\right\rangle = \sqrt{n_B + 1}\left|n_B + 1, n_F\right\rangle$$

Suppersymmetric operators

Annihilation and creation operators:

$$b\left|n_B,n_F\right\rangle=\sqrt{n_B}\left|n_B-1,n_F\right\rangle,\ b^\dagger\left|n_B,n_F\right\rangle=\sqrt{n_B+1}\left|n_B+1,n_F\right\rangle$$

$$\left[b,b^\dagger\right]=1,\ b^\dagger b=\hat{N}_B$$

Suppersymmetric operators

Annihilation and creation operators:

$$b\,|n_B, n_F\rangle = \sqrt{n_B}\,|n_B - 1, n_F\rangle\,,\ \ b^\dagger\,|n_B, n_F\rangle = \sqrt{n_B + 1}\,|n_B + 1, n_F\rangle$$

$$\left[b, b^\dagger\right] = 1,\ \ b^\dagger b = \hat{N}_B$$

$$f\,|n_B, n_F\rangle = \sqrt{n_F}\,|n_B, n_F - 1\rangle\,,\ \ f^\dagger\,|n_B, n_F\rangle = \sqrt{n_F + 1}\,|n_B, n_F + 1\rangle$$

Suppersymmetric operators

Annihilation and creation operators:

$$b\left|n_B, n_F\right\rangle = \sqrt{n_B}\left|n_B - 1, n_F\right\rangle, \ b^\dagger\left|n_B, n_F\right\rangle = \sqrt{n_B + 1}\left|n_B + 1, n_F\right\rangle$$

$$\left[b, b^\dagger\right] = 1, \ b^\dagger b = \hat{N}_B$$

$$f\left|n_B, n_F\right\rangle = \sqrt{n_F}\left|n_B, n_F - 1\right\rangle, \ f^\dagger\left|n_B, n_F\right\rangle = \sqrt{n_F + 1}\left|n_B, n_F + 1\right\rangle$$

$$\left\{f, f^\dagger\right\} = ff^\dagger + f^\dagger f = 1, \ f^\dagger f = \hat{N}_F$$

Suppersymmetric operators

Annihilation and creation operators:

$$b\left|n_B,n_F\right\rangle=\sqrt{n_B}\left|n_B-1,n_F\right\rangle,\ b^\dagger\left|n_B,n_F\right\rangle=\sqrt{n_B+1}\left|n_B+1,n_F\right\rangle$$

$$\left[b,b^\dagger\right]=1,\ b^\dagger b=\hat{N}_B$$

$$f\left|n_B,n_F\right\rangle=\sqrt{n_F}\left|n_B,n_F-1\right\rangle,\ f^\dagger\left|n_B,n_F\right\rangle=\sqrt{n_F+1}\left|n_B,n_F+1\right\rangle$$

$$\left\{f,f^\dagger\right\}=ff^\dagger+f^\dagger f=1,\ f^\dagger f=\hat{N}_F$$

$$Q_+=bf^\dagger\ \Rightarrow\ Q_+\left|n_B,n_F\right\rangle\sim\left|n_B-1,n_F+1\right\rangle$$

$$Q_-=b^\dagger f\ \Rightarrow\ Q_-\left|n_B,n_F\right\rangle\sim\left|n_B+1,n_F-1\right\rangle$$

Suppersymmetric operators (2)

$$Q_+ = bf^\dagger \;\Rightarrow\; Q_+ \left|n_B, n_F\right\rangle \sim \left|n_B - 1, n_F + 1\right\rangle$$
$$Q_- = b^\dagger f \;\Rightarrow\; Q_- \left|n_B, n_F\right\rangle \sim \left|n_B + 1, n_F - 1\right\rangle$$

Suppersymmetric operators (2)

$$Q_+ = bf^\dagger \;\Rightarrow\; Q_+ \left|n_B, n_F\right\rangle \sim \left|n_B - 1, n_F + 1\right\rangle$$

$$Q_- = b^\dagger f \;\Rightarrow\; Q_- \left|n_B, n_F\right\rangle \sim \left|n_B + 1, n_F - 1\right\rangle$$

$$f^2 = \left(f^\dagger\right)^2 = 0 \;\Rightarrow\; Q_+^2 = Q_-^2 = 0$$

Suppersymmetric operators (2)

$$Q_+ = bf^\dagger \;\Rightarrow\; Q_+ |n_B, n_F\rangle \sim |n_B - 1, n_F + 1\rangle$$

$$Q_- = b^\dagger f \;\Rightarrow\; Q_- |n_B, n_F\rangle \sim |n_B + 1, n_F - 1\rangle$$

$$f^2 = \left(f^\dagger\right)^2 = 0 \;\Rightarrow\; Q_+^2 = Q_-^2 = 0$$

$$Q_1 = Q_+ + Q_-,\; Q_2 = -i(Q_+ - Q_-) \;\Rightarrow \{Q_1, Q_2\} = 0$$

Suppersymmetric operators (2)

$$Q_+ = bf^\dagger \;\Rightarrow\; Q_+ \left|n_B, n_F\right\rangle \sim \left|n_B - 1, n_F + 1\right\rangle$$

$$Q_- = b^\dagger f \;\Rightarrow\; Q_- \left|n_B, n_F\right\rangle \sim \left|n_B + 1, n_F - 1\right\rangle$$

$$f^2 = \left(f^\dagger\right)^2 = 0 \;\Rightarrow\; Q_+^2 = Q_-^2 = 0$$

$$Q_1 = Q_+ + Q_-,\; Q_2 = -i(Q_+ - Q_-) \;\Rightarrow \{Q_1, Q_2\} = 0$$

Consider $\hat{H} = \{Q_+, Q_-\} = Q_1^2 = Q_2^2$ (Simplest Hamiltonian)

Suppersymmetric operators (2)

$$Q_+ = bf^\dagger \;\Rightarrow\; Q_+ |n_B, n_F\rangle \sim |n_B - 1, n_F + 1\rangle$$

$$Q_- = b^\dagger f \;\Rightarrow\; Q_- |n_B, n_F\rangle \sim |n_B + 1, n_F - 1\rangle$$

$$f^2 = \left(f^\dagger\right)^2 = 0 \;\Rightarrow\; Q_+^2 = Q_-^2 = 0$$

$$Q_1 = Q_+ + Q_-,\; Q_2 = -i(Q_+ - Q_-) \;\Rightarrow \{Q_1, Q_2\} = 0$$

Consider $\hat{H} = \{Q_+, Q_-\} = Q_1^2 = Q_2^2$ (Simplest Hamiltonian)

$[H, Q] = 0$, where $Q = Q_+, Q_-, Q_1$ or Q_2

Suppersymmetric operators (2)

$$Q_+ = bf^\dagger \;\Rightarrow\; Q_+ \left|n_B, n_F\right\rangle \sim \left|n_B - 1, n_F + 1\right\rangle$$

$$Q_- = b^\dagger f \;\Rightarrow\; Q_- \left|n_B, n_F\right\rangle \sim \left|n_B + 1, n_F - 1\right\rangle$$

$$f^2 = \left(f^\dagger\right)^2 = 0 \;\Rightarrow\; Q_+^2 = Q_-^2 = 0$$

$$Q_1 = Q_+ + Q_-,\; Q_2 = -i(Q_+ - Q_-) \;\Rightarrow \{Q_1, Q_2\} = 0$$

Consider $\hat{H} = \{Q_+, Q_-\} = Q_1^2 = Q_2^2$ (Simplest Hamiltonian)

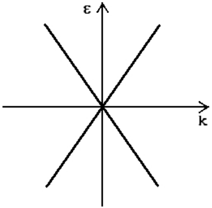
$[H, Q] = 0$, where $Q = Q_+, Q_-, Q_1$ or Q_2

Therefore the values of these operators are conserved

Generalizing Hamiltonian

Back to our Hamiltonian

$$\hat{H} = \begin{pmatrix} 0 & v\pi_- \\ v\pi_+ & 0 \end{pmatrix}$$



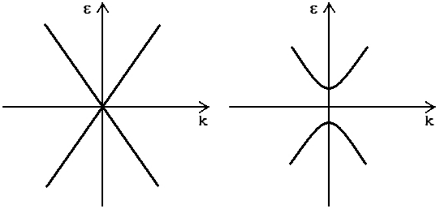
Generalizing Hamiltonian

Back to our Hamiltonian

$$\hat{H} = \begin{pmatrix} 0 & v\pi_- \\ v\pi_+ & 0 \end{pmatrix}$$

We introduce the mass term Δ:

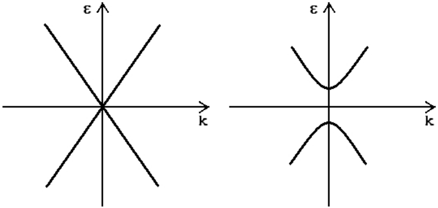
$$\hat{H} = \begin{pmatrix} \Delta & v\pi_- \\ v\pi_+ & -\Delta \end{pmatrix}$$



Generalizing Hamiltonian

Back to our Hamiltonian

$$\hat{H} = \begin{pmatrix} 0 & v\pi_- \\ v\pi_+ & 0 \end{pmatrix}$$

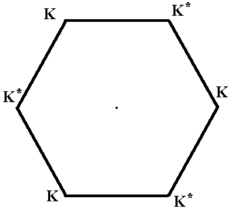


We introduce the mass term Δ:

$$\hat{H} = \begin{pmatrix} \Delta & v\pi_- \\ v\pi_+ & -\Delta \end{pmatrix}$$

Recall: there are two independent points in momentum space: K and K*. They act in different subspaces, so the total Hamiltonian has the form

$$\begin{pmatrix} \hat{H}_K & 0 \\ 0 & \hat{H}_{K^*} \end{pmatrix}$$



Suppersymmetric operators

Summarizing Hamiltonian around K and K^* we get

$$\hat{H} = \begin{pmatrix} \Delta & v\pi_- & & \\ v\pi_+ & -\Delta & & \\ & & \Delta & v\pi_+ \\ & & v\pi_- & -\Delta \end{pmatrix}$$

Suppersymmetric operators

Summarizing Hamiltonian around K and K^* we get

$$\hat{H} = \begin{pmatrix} \Delta & v\pi_- & & \\ v\pi_+ & -\Delta & & \\ & & \Delta & v\pi_+ \\ & & v\pi_- & -\Delta \end{pmatrix}$$

$$= \begin{pmatrix} \Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v & 0 \\ 0 & \Delta\sigma_z + (\sigma_+\pi_+ + \sigma_-\pi_-)v \end{pmatrix}$$

Suppersymmetric operators

Summarizing Hamiltonian around K and K^* we get

$$\hat{H} = \begin{pmatrix} \Delta & v\pi_- & & \\ v\pi_+ & -\Delta & & \\ & & \Delta & v\pi_+ \\ & & v\pi_- & -\Delta \end{pmatrix}$$

$$= \begin{pmatrix} \Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v & 0 \\ 0 & \Delta\sigma_z + (\sigma_+\pi_+ + \sigma_-\pi_-)v \end{pmatrix}$$

If we interchange the last two basis vectors we get

$$\hat{H} = \begin{pmatrix} \Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v & 0 \\ 0 & -\Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v \end{pmatrix}$$

Squared Hamiltonian

$$\hat{H} = \begin{pmatrix} \Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v & 0 \\ 0 & -\Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v \end{pmatrix}$$

Squared Hamiltonian

$$\hat{H} = \begin{pmatrix} \Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v & 0 \\ 0 & -\Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v \end{pmatrix}$$

Let us look at the square of the Hamiltonian around K:

Squared Hamiltonian

$$\hat{H} = \begin{pmatrix} \Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v & 0 \\ 0 & -\Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v \end{pmatrix}$$

Let us look at the square of the Hamiltonian around K:

$$(\Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v)^2 = (\Delta\sigma_z)^2 + ((\sigma_+\pi_- + \sigma_-\pi_+)v)^2 +$$

Squared Hamiltonian

$$\hat{H} = \begin{pmatrix} \Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v & 0 \\ 0 & -\Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v \end{pmatrix}$$

Let us look at the square of the Hamiltonian around K:

$$(\Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v)^2 = (\Delta\sigma_z)^2 + ((\sigma_+\pi_- + \sigma_-\pi_+)v)^2 +$$
$$+ v\Delta\left\{\sigma_z, \vec{\sigma}_\perp\right\}\hat{\vec{\pi}}_\perp = (\Delta\sigma_z)^2 + ((\sigma_+\pi_- + \sigma_-\pi_+)v)^2$$

Since $\left\{\sigma_z, \vec{\sigma}_\perp\right\} = 0$

Squared Hamiltonian

$$\hat{H} = \begin{pmatrix} \Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v & 0 \\ 0 & -\Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v \end{pmatrix}$$

Let us look at the square of the Hamiltonian around K:

$$(\Delta\sigma_z + (\sigma_+\pi_- + \sigma_-\pi_+)v)^2 = (\Delta\sigma_z)^2 + ((\sigma_+\pi_- + \sigma_-\pi_+)v)^2 +$$

$$+ v\Delta \left\{\sigma_z, \vec{\sigma}_\perp\right\} \hat{\vec{\pi}}_\perp = (\Delta\sigma_z)^2 + ((\sigma_+\pi_- + \sigma_-\pi_+)v)^2$$

$$\text{Since } \left\{\sigma_z, \vec{\sigma}_\perp\right\} = 0$$

$\hat{H}^2 = (\Delta\sigma_z)^2 + ((\sigma_+\pi_- + \sigma_-\pi_+)v)^2$ contains therefore both solutions for K and K*

Squared Hamiltonian

$$\hat{H}^2\psi = \varepsilon^2\psi = (\Delta\sigma_z + v\sigma_\perp\pi_\perp)^2\psi = \Delta^2 + v^2(\sigma_\perp\pi_\perp)^2\psi$$

Squared Hamiltonian

$$\hat{H}^2\psi = \varepsilon^2\psi = (\Delta\sigma_z + v\sigma_\perp\pi_\perp)^2\psi = \Delta^2 + v^2(\sigma_\perp\pi_\perp)^2\psi$$

$$(\vec{\sigma}_\perp\hat{\vec{\pi}}_\perp)^2 = \frac{1}{2}\left[\{\hat{\pi_+}, \hat{\pi_-}\} - \gamma^2 \mathit{sgn}(eB)\sigma_z\right]$$

Squared Hamiltonian

$$\hat{H}^2\psi = \varepsilon^2\psi = (\Delta\sigma_z + v\sigma_\perp\pi_\perp)^2\psi = \Delta^2 + v^2(\sigma_\perp\pi_\perp)^2\psi$$

$$(\vec{\sigma}_\perp\hat{\vec{\pi}}_\perp)^2 = \frac{1}{2}\left[\{\hat{\pi_+}, \hat{\pi_-}\} - \gamma^2 \mathit{sgn}(eB)\sigma_z\right]$$

$$\{\hat{\pi_+}, \hat{\pi_-}\}\left|n\right\rangle = \gamma^2(2n+1)\left|n\right\rangle \quad \text{always}$$

Squared Hamiltonian

$$\hat{H}^2\psi = \varepsilon^2\psi = (\Delta\sigma_z + v\sigma_\perp\pi_\perp)^2\psi = \Delta^2 + v^2(\sigma_\perp\pi_\perp)^2\psi$$

$$(\vec{\sigma}_\perp\hat{\vec{\pi}}_\perp)^2 = \frac{1}{2}\left[\{\hat{\pi_+}, \hat{\pi_-}\} - \gamma^2 \mathit{sgn}(eB)\sigma_z\right]$$

$$\{\hat{\pi_+}, \hat{\pi_-}\}|n\rangle = \gamma^2(2n+1)|n\rangle \quad \text{always}$$

$$(\sigma_\perp\pi_\perp)^2|n\rangle|\alpha\rangle = \frac{1}{2}\gamma^2\left[2n+1-\mathit{sgn}(eB)\sigma_z\right]|n\rangle|\alpha\rangle$$

Squared Hamiltonian

$$\hat{H}^2\psi = \varepsilon^2\psi = (\Delta\sigma_z + v\sigma_\perp\pi_\perp)^2\psi = \Delta^2 + v^2(\sigma_\perp\pi_\perp)^2\psi$$

$$(\vec{\sigma}_\perp\hat{\vec{\pi}}_\perp)^2 = \frac{1}{2}\left[\{\hat{\pi_+}, \hat{\pi_-}\} - \gamma^2 \mathit{sgn}(eB)\sigma_z\right]$$

$$\{\hat{\pi_+}, \hat{\pi_-}\}\left|n\right\rangle = \gamma^2(2n+1)\left|n\right\rangle \quad \text{always}$$

$$(\sigma_\perp\pi_\perp)^2\left|n\right\rangle\left|\alpha\right\rangle = \frac{1}{2}\gamma^2\left[2n + 1 - \mathit{sgn}(eB)\sigma_z\right]\left|n\right\rangle\left|\alpha\right\rangle$$

Landau levels correspond to bosons, quasi-spin to fermions

Squared Hamiltonian

$$\hat{H}^2\psi = \varepsilon^2\psi = (\Delta\sigma_z + v\sigma_\perp\pi_\perp)^2\psi = \Delta^2 + v^2(\sigma_\perp\pi_\perp)^2\psi$$

$$(\vec{\sigma}_\perp\hat{\vec{\pi}}_\perp)^2 = \frac{1}{2}\left[\{\hat{\pi_+}, \hat{\pi_-}\} - \gamma^2 \mathit{sgn}(eB)\sigma_z\right]$$

$$\{\hat{\pi_+}, \hat{\pi_-}\}|n\rangle = \gamma^2(2n+1)|n\rangle \quad \text{always}$$

$$(\sigma_\perp\pi_\perp)^2|n\rangle|\alpha\rangle = \frac{1}{2}\gamma^2\left[2n+1-\mathit{sgn}(eB)\sigma_z\right]|n\rangle|\alpha\rangle$$

Landau levels correspond to bosons, quasi-spin to fermions

$$\varepsilon^2 = \Delta^2 + \frac{v^2\gamma^2}{2}(2n+1-\mathit{sgn}(eB)\sigma_z)$$

$$\varepsilon^2_{n,\downarrow} = \varepsilon^2_{n-1,\uparrow} = \Delta^2 + v^2\gamma^2 n \quad (\text{provided } eB < 0)$$

Basics of Quantum Hall effect

- Quantum-mechanical version of the Hall effect

Basics of Quantum Hall effect

- Quantum-mechanical version of the Hall effect
- Observed in 2D electron systems

Basics of Quantum Hall effect

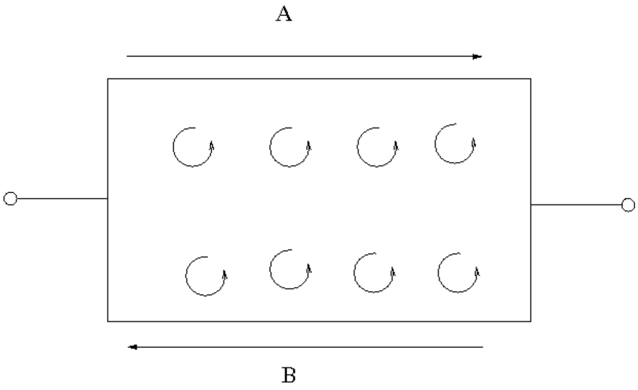
- Quantum-mechanical version of the Hall effect
- Observed in 2D electron systems
- Low temperatures and strong magnetic fields

Basics of Quantum Hall effect

- Quantum-mechanical version of the Hall effect
- Observed in 2D electron systems
- Low temperatures and strong magnetic fields
- Hall conductance takes on quantized values $\sigma = \nu \, \frac{e^2}{h}$

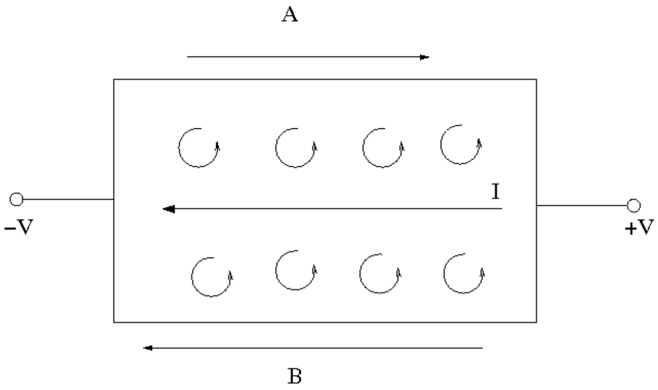
Quantum Hall effect

$V_{AB} = 0$



Quantum Hall effect(2)

$V_{AB} \neq 0$



Quantum Hall effect(3)

$$\sigma_{xy} = -\frac{ec\rho}{B}, \quad \rho = n_e - n_h$$

Quantum Hall effect(3)

$$\sigma_{xy} = -\frac{ec\rho}{B}, \quad \rho = n_e - n_h$$

filling factor: $\nu_B = \dfrac{|\rho|}{N_B}, \quad \nu_B = \dfrac{\pi c\hbar}{|eB|}\,|\rho|$

Recall: $N_B = \dfrac{1}{2\pi l_H^2} \times 2$

Quantum Hall effect(3)

$$\sigma_{xy} = -\frac{ec\rho}{B}, \quad \rho = n_e - n_h$$

filling factor: $\nu_B = \frac{|\rho|}{N_B}, \quad \nu_B = \frac{\pi c\hbar}{|eB|}|\rho|$

Recall: $N_B = \frac{1}{2\pi l_H^2} \times 2$

$$\Rightarrow \sigma_{xy} = -\frac{e^2 c}{|eB|}|\rho| \textsf{sgn}(eB)\textsf{sgn}(\mu) = -\frac{e^2}{\pi\hbar}\nu_B \textsf{sgn}(eB)\textsf{sgn}(\mu)$$

Unusual Quantum Hall effect

$$n_e = \frac{1}{e^{\frac{\varepsilon - \mu}{T}} + 1}, \quad n_h = 1 - n_e$$

Unusual Quantum Hall effect

$$n_e = \frac{1}{e^{\frac{\varepsilon-\mu}{T}}+1}, \quad n_h = 1 - n_e$$

$$\Rightarrow \; n_e - n_h = \tanh\frac{\mu-\varepsilon}{2T}$$

Unusual Quantum Hall effect

$$n_e = \frac{1}{e^{\frac{\varepsilon-\mu}{T}} + 1}, \quad n_h = 1 - n_e$$

$$\Rightarrow \; n_e - n_h = \tanh \frac{\mu - \varepsilon}{2T}$$

$$\nu_B \mathit{sgn}(\mu) = \frac{1}{2}\left(\tanh\frac{\mu+\Delta}{2T} + \tanh\frac{\mu-\Delta}{2T} + 2\sum_{n=1}^{\infty}\left(\tanh\frac{\mu+\varepsilon_n}{2T} + \tanh\frac{\mu-\varepsilon_n}{2T}\right)\right)$$

Unusual Quantum Hall effect

$$n_e = \frac{1}{e^{\frac{\varepsilon-\mu}{T}}+1}, \quad n_h = 1 - n_e$$

$$\Rightarrow \; n_e - n_h = \tanh\frac{\mu-\varepsilon}{2T}$$

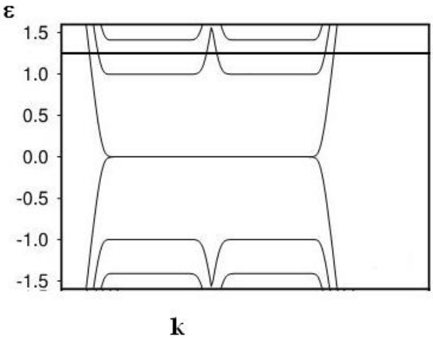
$$\nu_B \mathsf{sgn}(\mu) = \frac{1}{2}\left(\tanh\frac{\mu+\Delta}{2T} + \tanh\frac{\mu-\Delta}{2T} + 2\sum_{n=1}^{\infty}\left(\tanh\frac{\mu+\varepsilon_n}{2T} + \tanh\frac{\mu-\varepsilon_n}{2T}\right)\right)$$

$$\varepsilon_n = \sqrt{\Delta^2 + \gamma^2 v^2 n}$$

$$\tanh\left(\frac{\omega}{2T}\right) \to \mathsf{sgn}\omega \text{ for } T \to 0$$

Unusual Quantum Hall effect(2)

Intersections of the chemical potential with Landau-levels deformed near the edges correspond to the conducting electrons:



Unusual Quantum Hall effect(3)

Unusual first half-step of σ_{xy}: