Laboratory of Applied Mathematics and Mechanics

Modeling of contact interaction with friction

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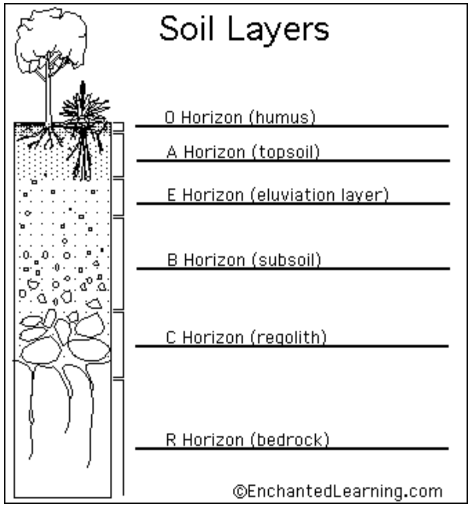
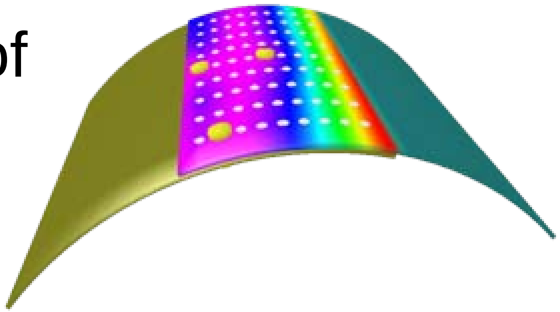
Objective

- Contact problems are highly nonlinear, their solution is always expensive in terms of time and resources



- Development of the algorithm based on the minimization of deformation energy of mechanical system

- Two main spheres of application: simulation of riveting process, modeling of sliding between soil layers



Contents

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- Rigidity matrix concept
- Computational procedure
- Applications
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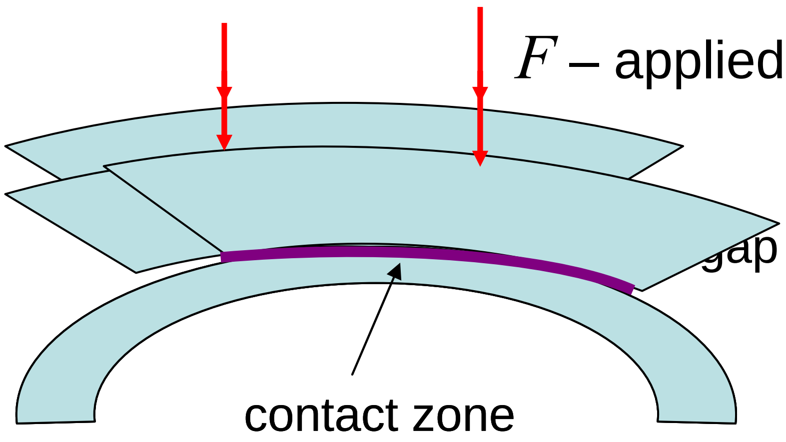
    - Sliding
    - Rode bending
- Future improvements

Problem statement

Two parts are initially separated, distance between them – initial gap



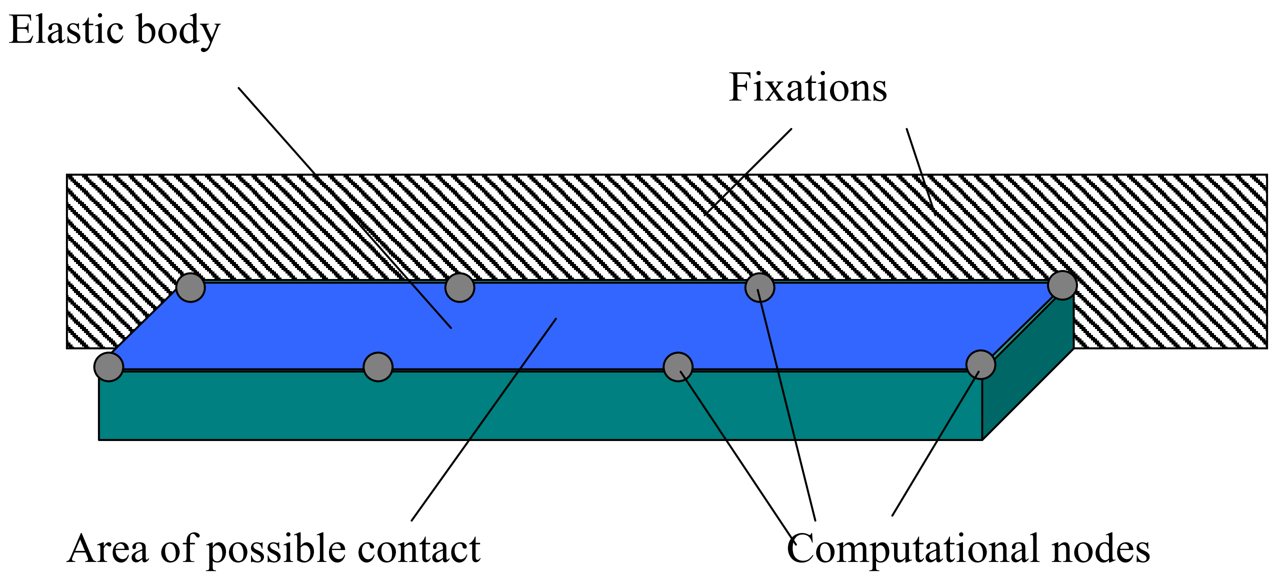
Unknown variables: U – vector of displacements in the contact zone

$U = \arg\min W$, where W – deformation energy of the mechanical system

Bodies may slide relative to each other according to the dry friction law

Rigidity matrix (1)

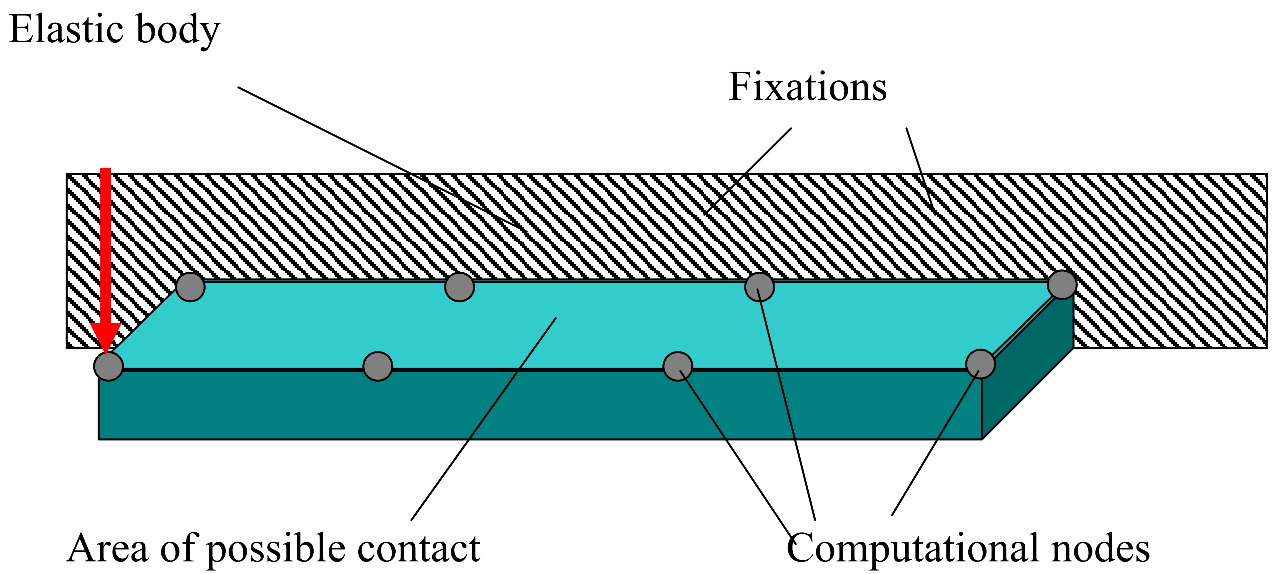
Deformation energy : $W = \frac{1}{2} U^T \cdot K \cdot U$, K - rigidity matrix



Thus, we substitute the body by its rigidity matrix, having the equality:

$$F = K \cdot U$$

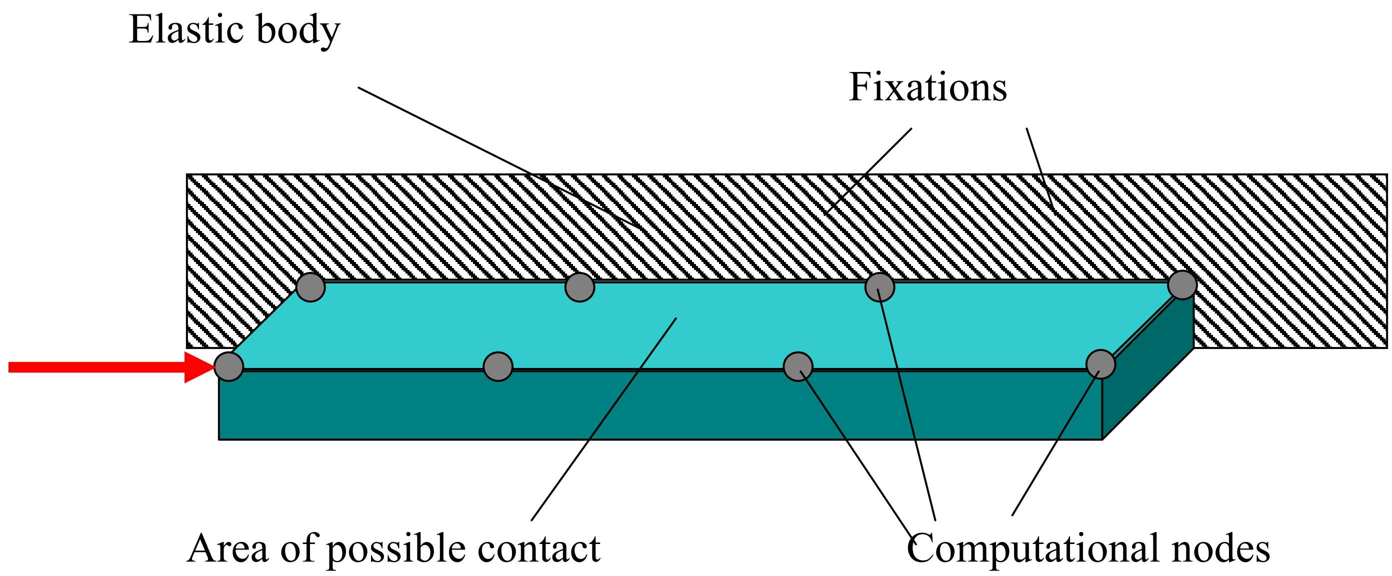
Rigidity matrix (2)



Loads can be applied in computational nodes in any direction

$$F^T = \left(f_{1x}, f_{1y}, f_{1z}, f_{2x}, f_{2y}, f_{2z}, \cdots, f_{nx}, f_{ny}, f_{nz}\right)$$

Rigidity matrix (2)



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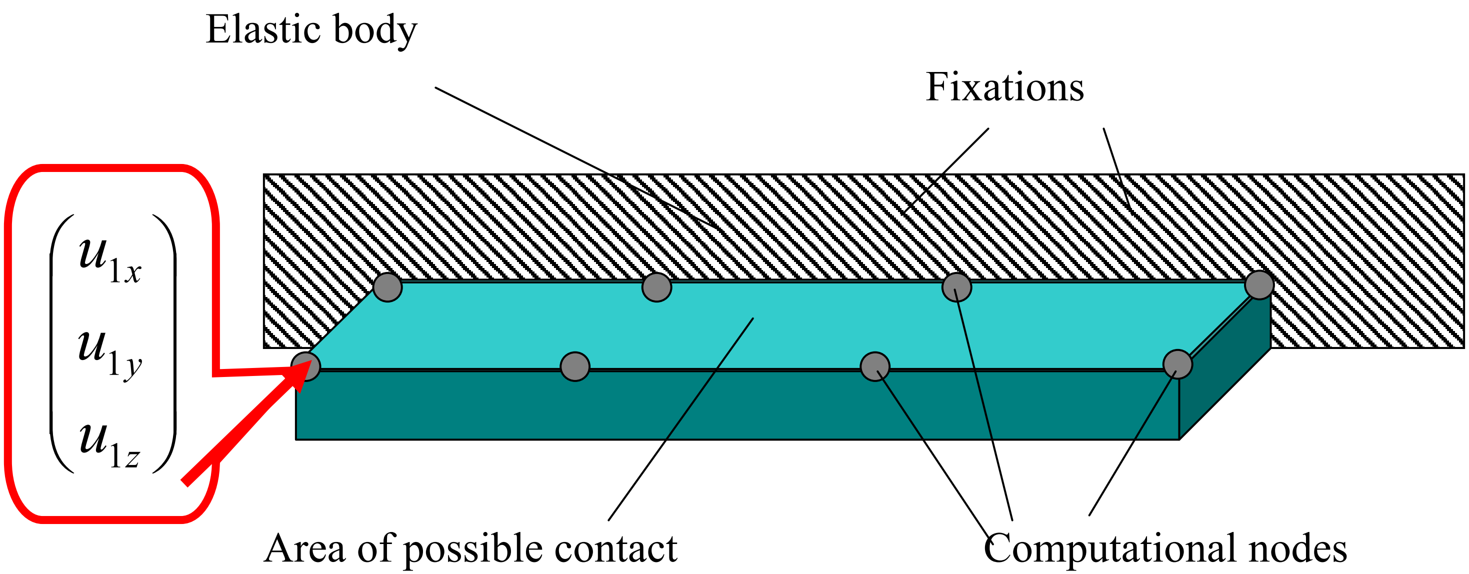


Displacements of nodes are calculated in all direction

$$U^T = \left(u_{1x}, u_{1y}, u_{1z}, u_{2x}, u_{2y}, u_{2z}, \cdots, u_{nx}, u_{ny}, u_{nz}\right)$$

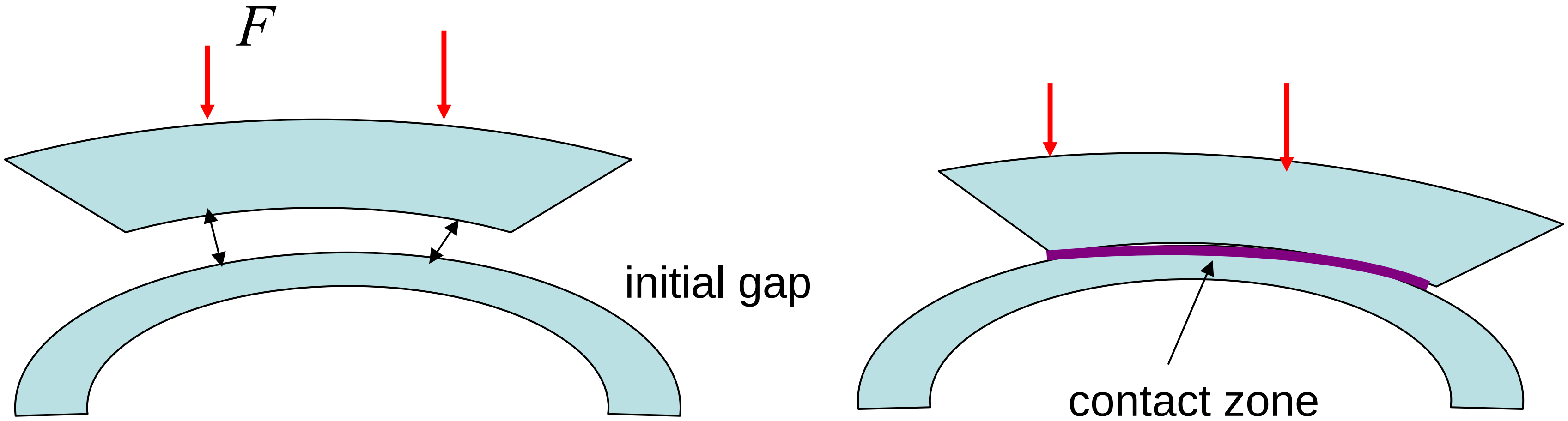
Main idea

Energy of deformation: $$W = \frac{1}{2} U^T \cdot K \cdot U \to \min$$

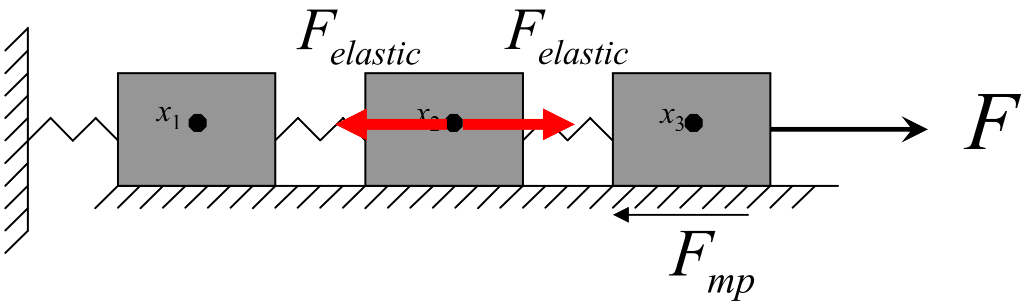
Forces acting on the system are balanced: $$K \cdot U = F$$

Displacements in normal direction are restricted: $$U_N \le \Delta$$

Forces in contact nodes should obey Coulomb law: $$|F_\tau| \le |\mu F_n|$$



One-dimensional problem



Energy of deformation: $W(x_1, x_2, \ldots x_N) = \frac{c}{2} \cdot U^T K U = \frac{c}{2} u_N^2 + c \sum_{i=1}^{N-1} u_i^2 - c \sum_{i=1}^{N} u_i u_{i-1}$

where c – spring rate, m – mass

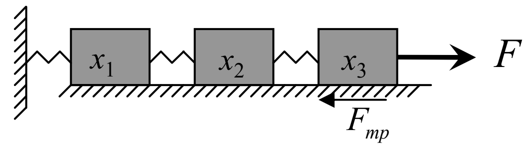
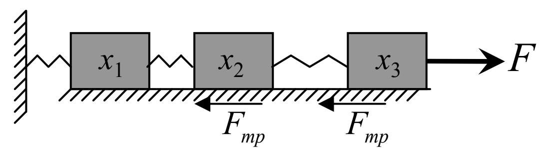
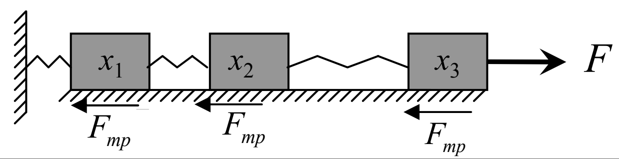
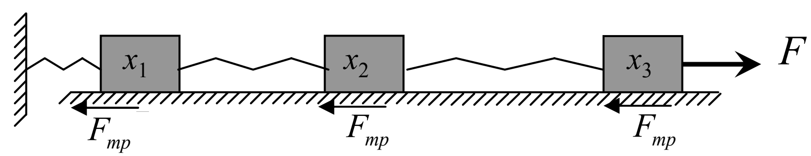
Friction force

Elastic force

$$-\mu mg \leq c(u_{i+1} - u_i) - c(u_i - u_{i-1}) \leq \mu mg, \quad i = 1, \ldots, N-1$$

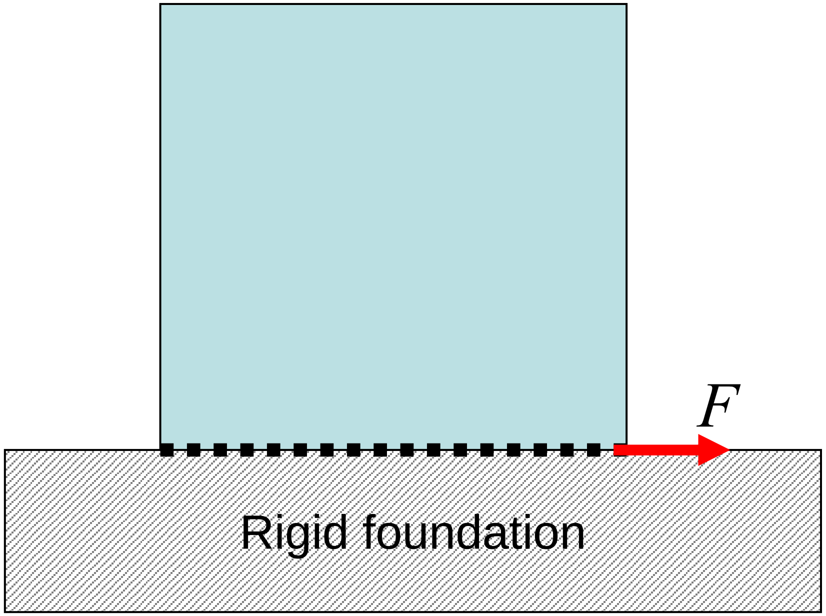
$$-\mu mg \leq F - c(u_N - u_{N-1}) \leq \mu mg$$

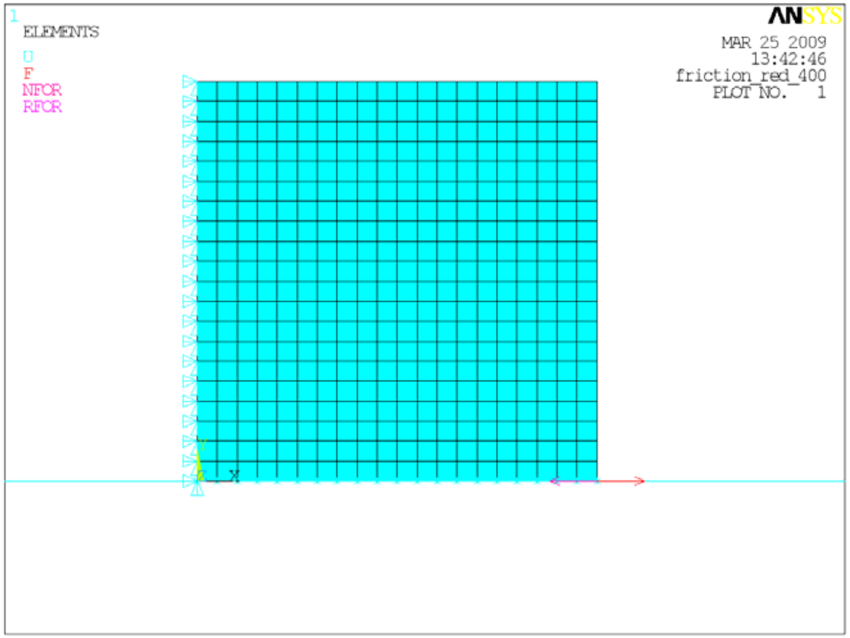
Results

№	F (Н)	x_1 (м)	x_2 (м)	x_3 (м)	Scheme of bodies displacements
1	3	0	0	0	
2	6	0	0	2	
3	9	0	1	6	
4	14	2	8	18	

2D sliding

- 2D deformable body
- Made of isotropic material
- Moves on the rigid foundation
- Gap between bodies initially is closed

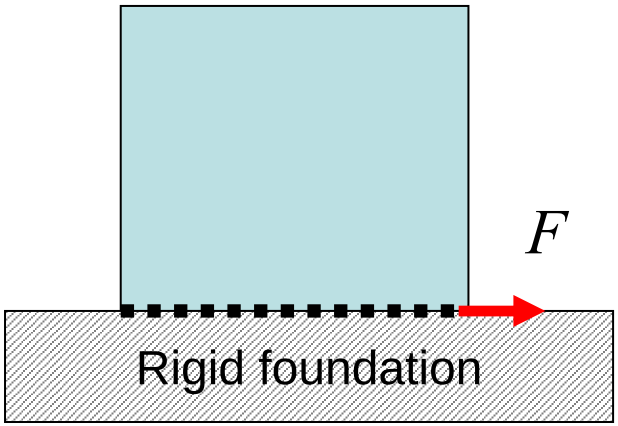




2D sliding

- Compute rigidity matrix by mentioned procedure

$$W = \frac{1}{2} U^T \cdot K \cdot U \to \min$$

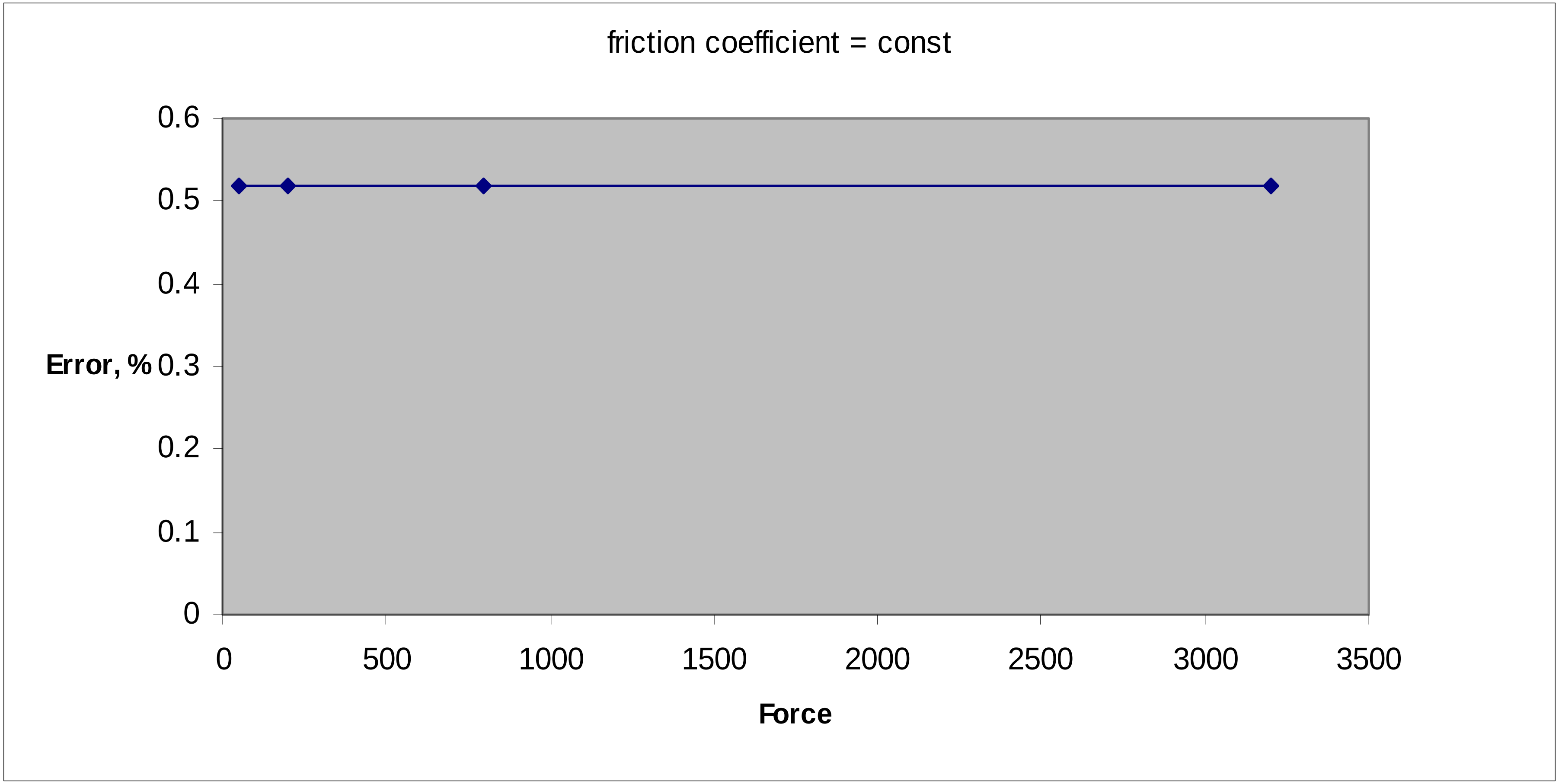


- All the nodes are contact ones:
- Gap is zero: $U_y = 0$

- Couloumb-Mohr law is to be satisfied in each node: $|F_\tau| \le |\mu F_n|$

Results comparison

Relative error : $\varepsilon = \dfrac{\max\limits_{i=1,n} \left| U^{calc} - U^{ANSYS} \right|}{\max\limits_{i=1,n} \left| U^{ANSYS} \right|}$



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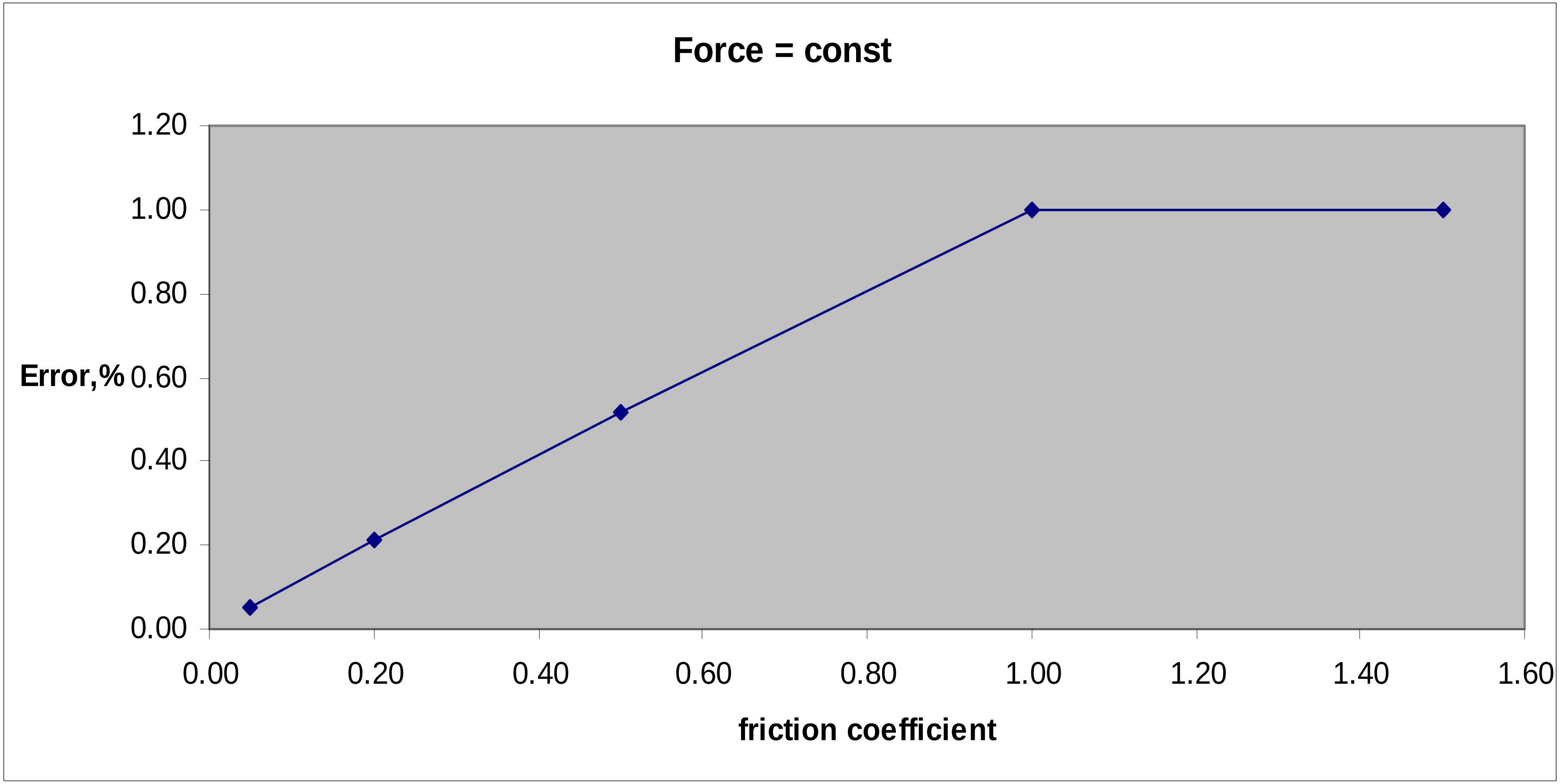
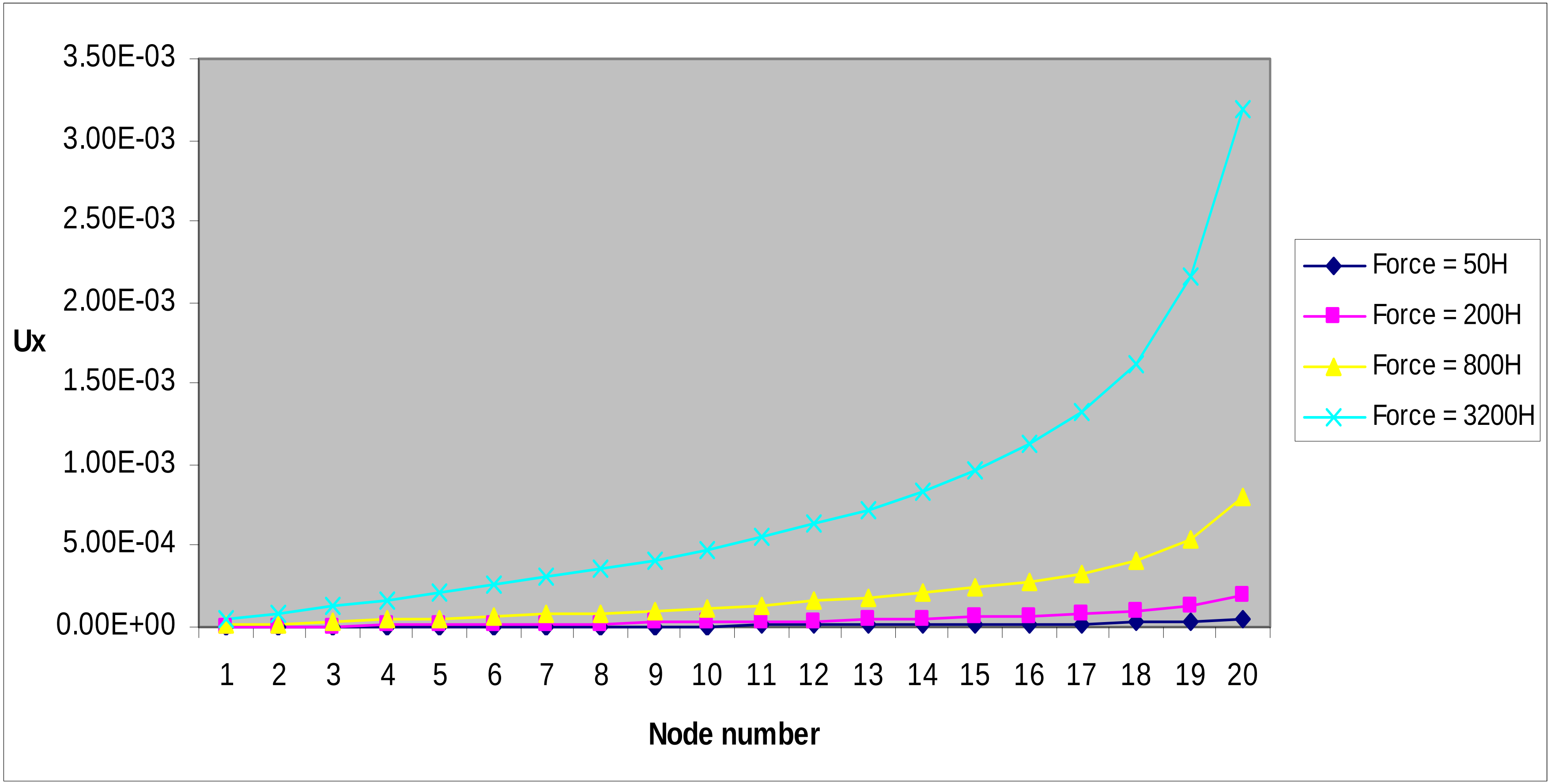


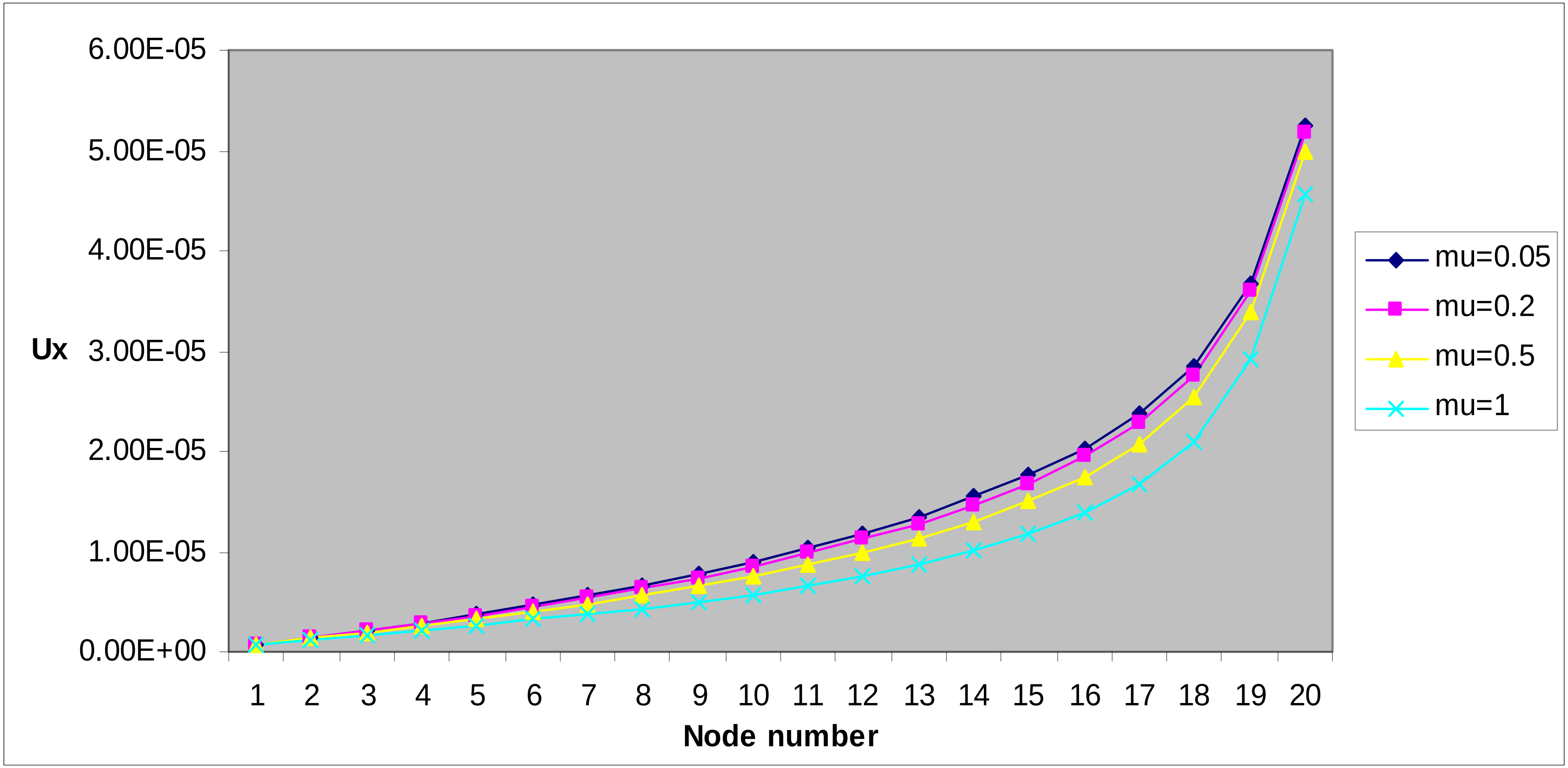
Results comparison

Dependence of displacements in X-direction on the force value



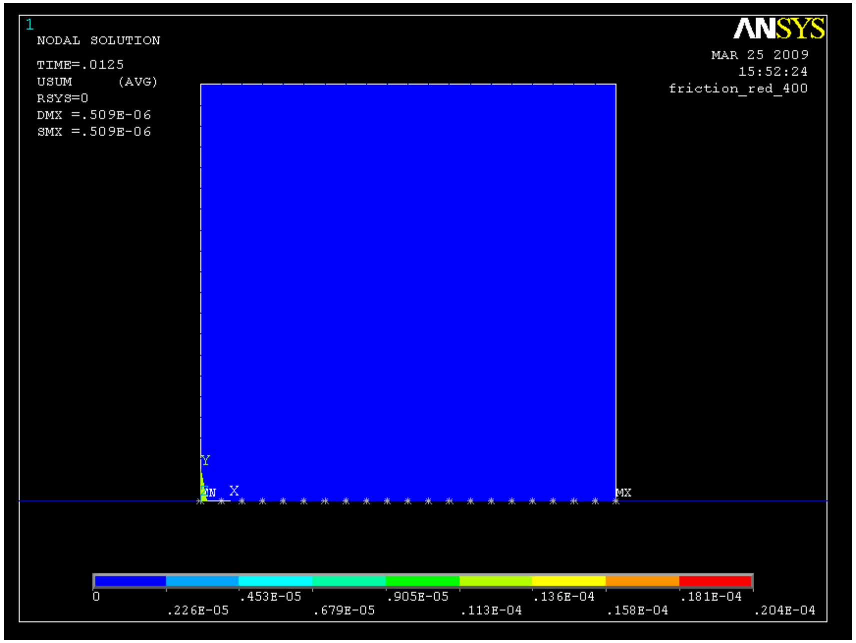
Results comparison

Dependence of displacements in X-direction on the value of friction coefficient

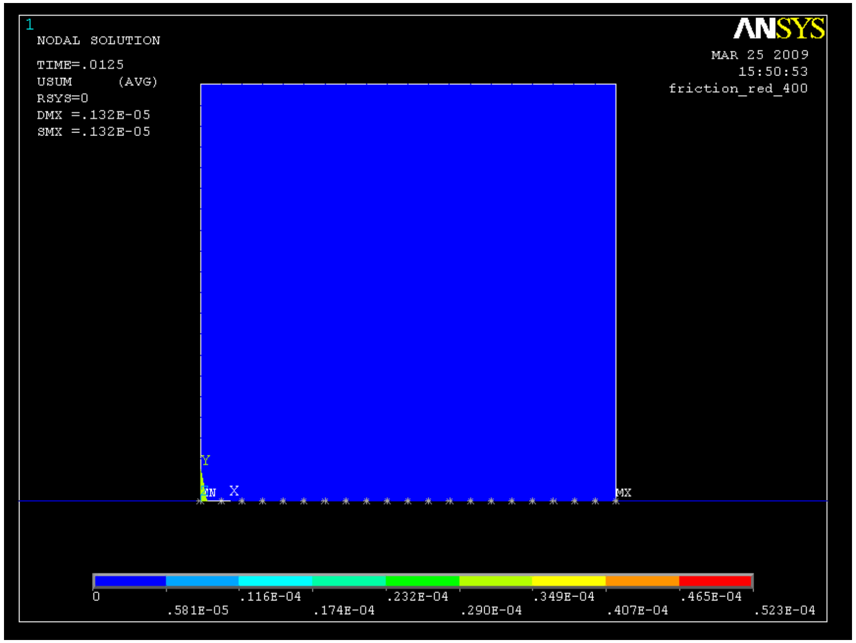


2D sliding

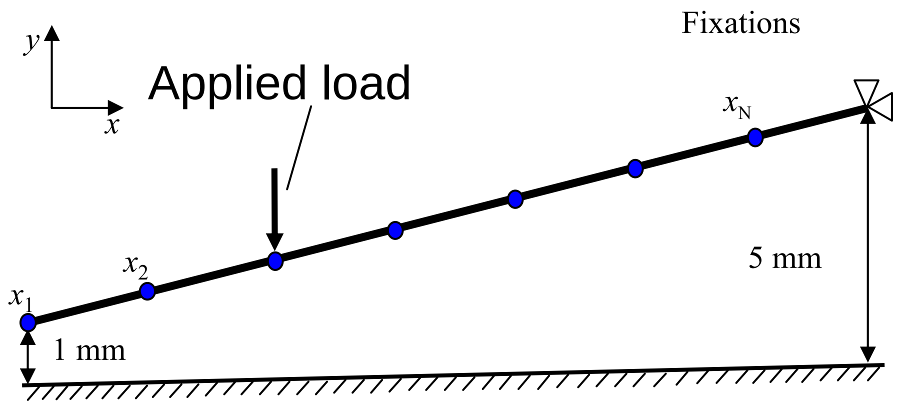
Body deformation for mu = 1



Body deformation for mu = 0.1

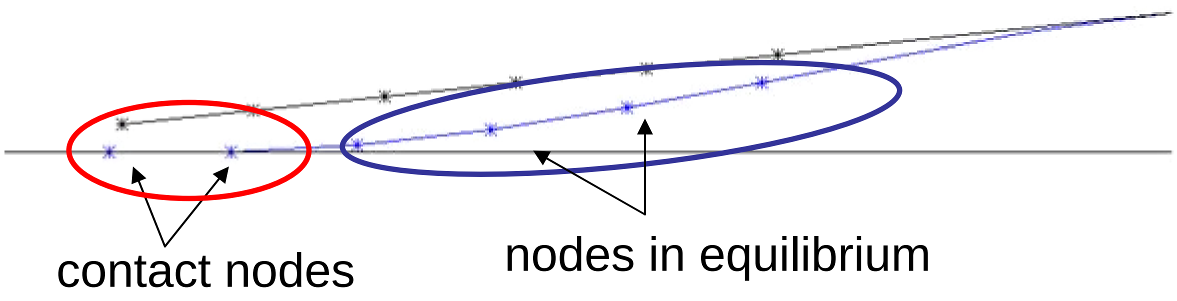


Rode bending



- Contact problem → Gap elimination; Calculation of sliding distances
- Zone of possible contact depends on applied loads
- It is necessary to derive iterative procedure

Iterative procedure



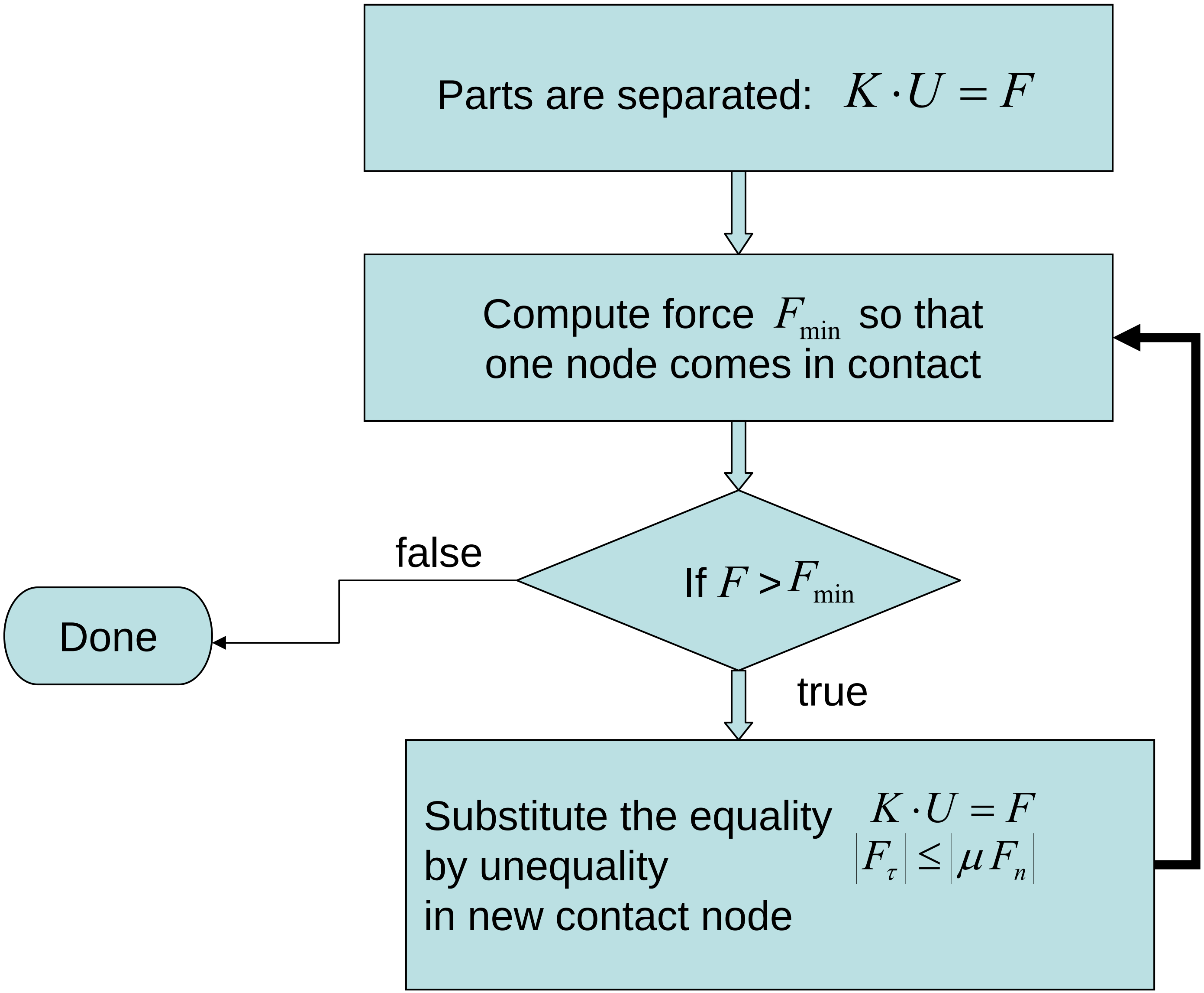
Equilibrium equations: $K \cdot U = F$

Unknown forces appear: $K \cdot U = F + f_{contact}$,

$f_{contact}$ - reaction force or friction force

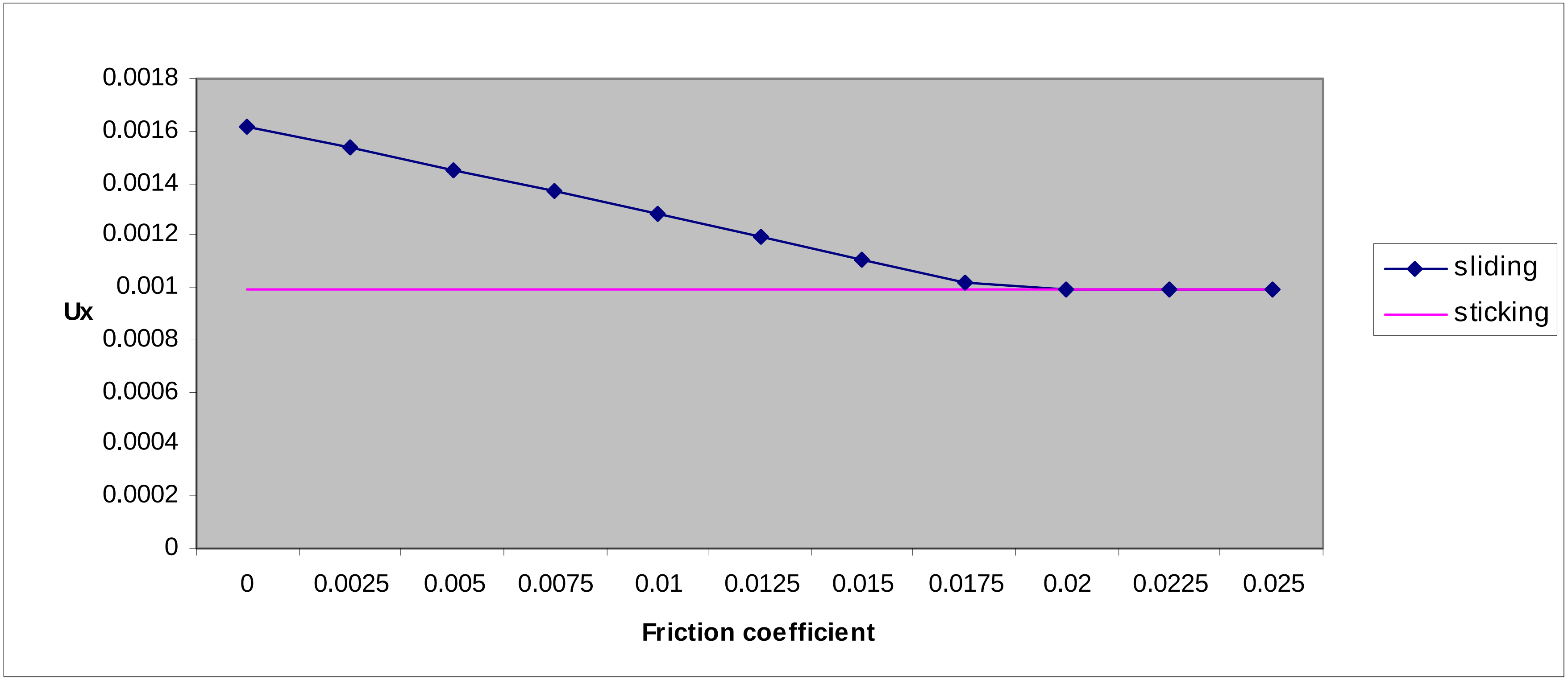
Coulomb-Mohr law: $|F_\tau| \le |\mu F_n|$

Iterative procedure



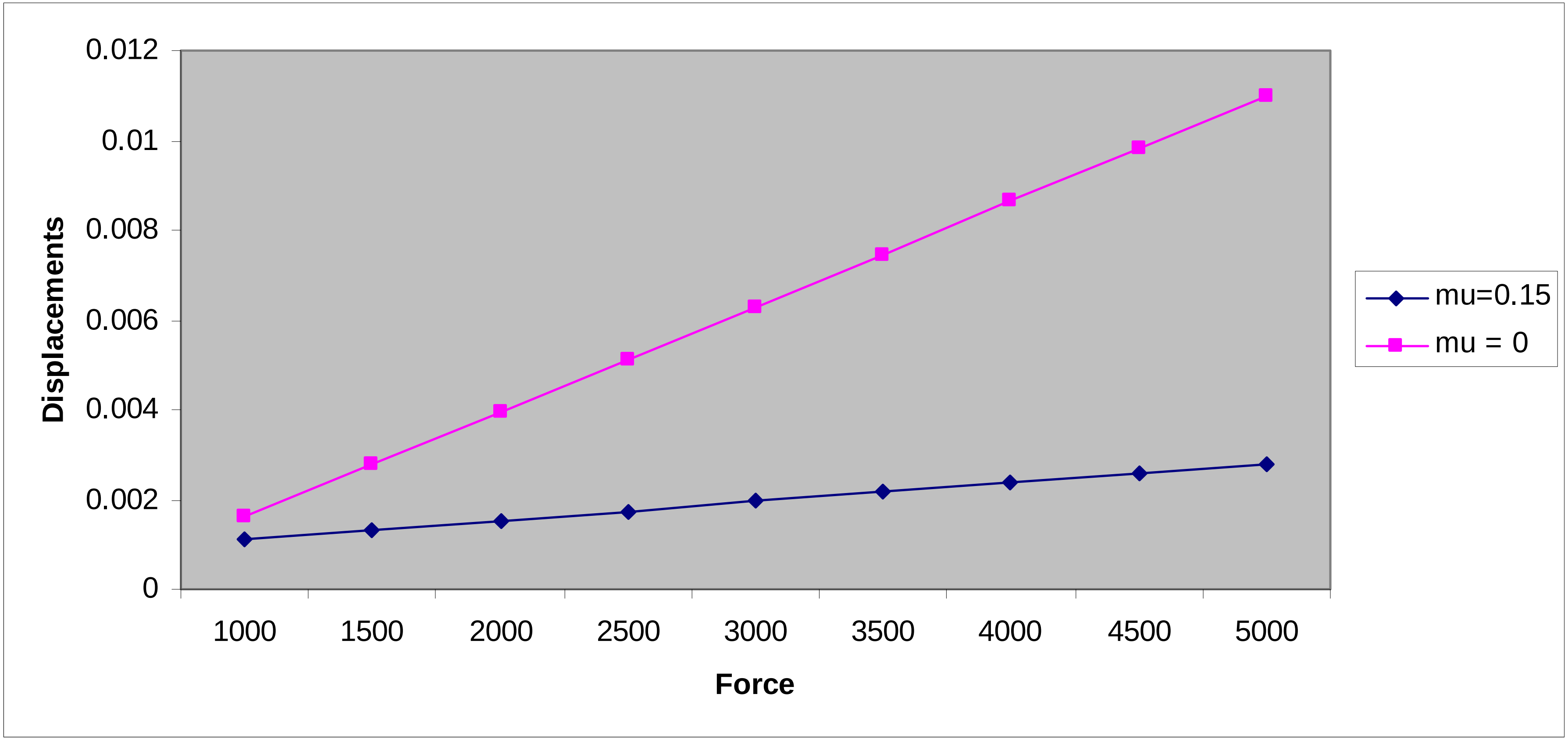
Results

Dependence of displacement in X-direction on the friction coefficient value in the last node



Results

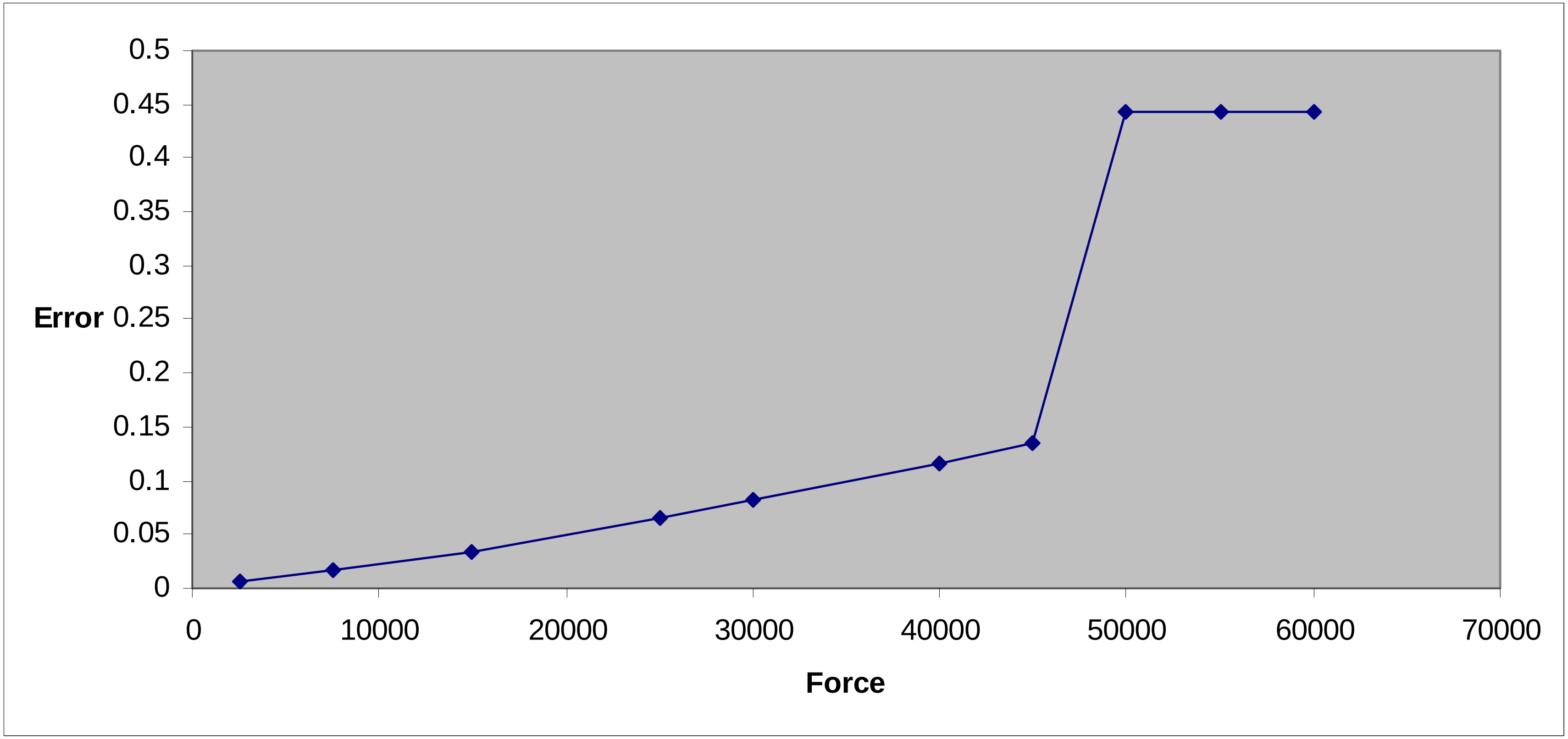
Dependence of displacement in X-direction on the force value in the last node



Results

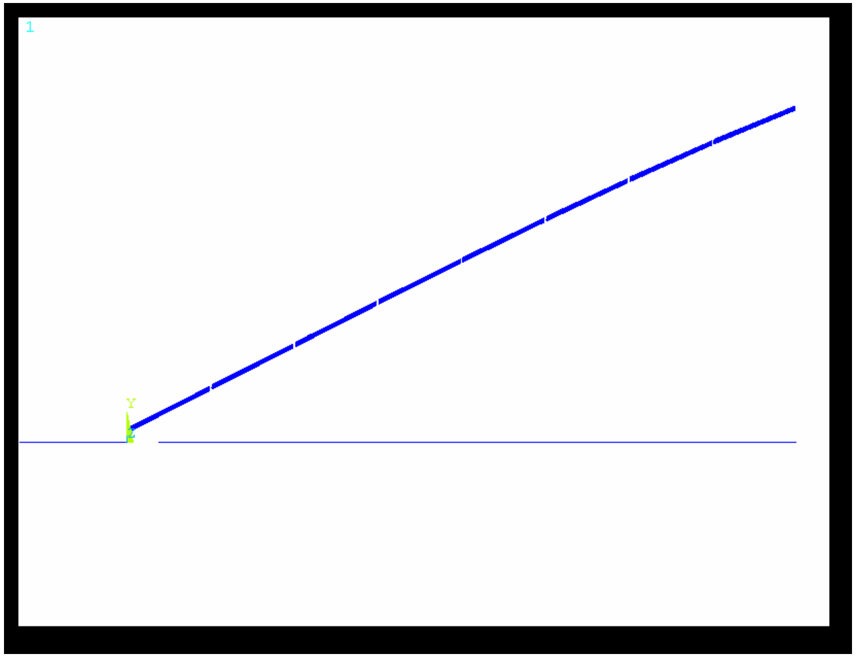
Dependence of relative error on the force value

Relative error : $\varepsilon = \dfrac{\max\limits_{i=1,n} \left| U^{calc} - U^{ANSYS} \right|}{\max\limits_{i=1,n} \left| U^{ANSYS} \right|}$

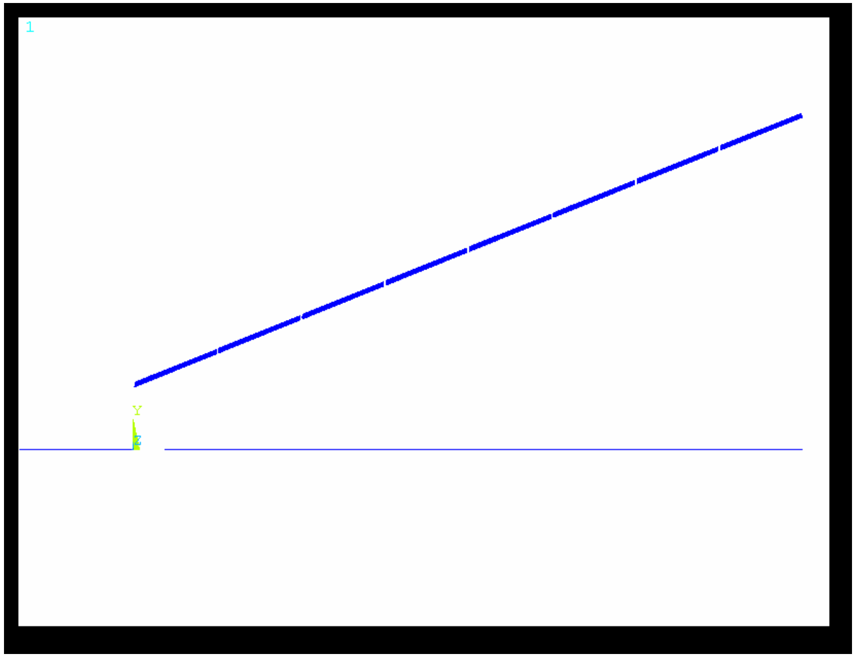


Deformation shapes

Two contact nodes



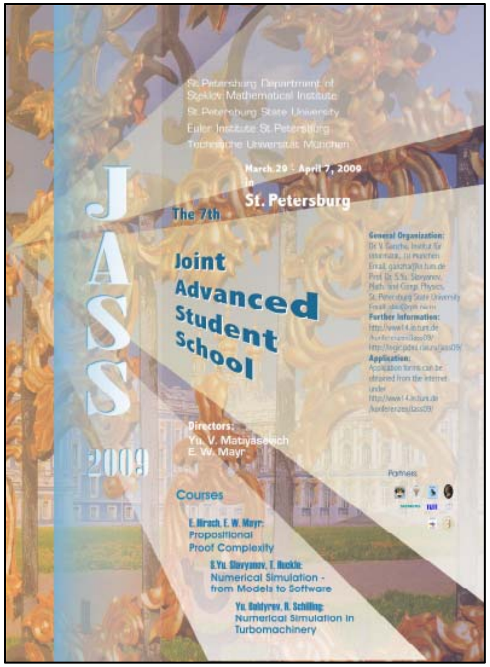
One contact node



Future improvements and investigations

- Sliding: - modeling two flexible bodies interaction
  - modeling systems with refined meshes
  - 3D geometry simulation

- Riveting process simulation:
  - modeling 2D and 3D objects
  - optimizing the procedure of defining contact zone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Thank you!