Formation Control for Multi-Robot Systems

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Overview

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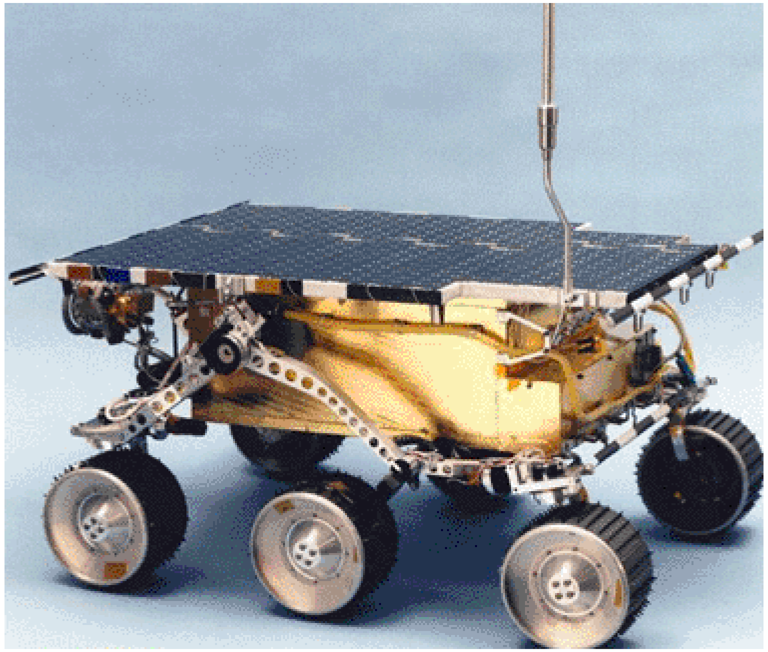
Introduction

- With the rapid development of artificial intelligence autonomous robots, sensors, automatic control, and computer vision systems, multi-robot systems have been widely applied to various real-world applications such as search and rescue, surveillance, collaborative exploration, and exploration of large and unmapped areas.



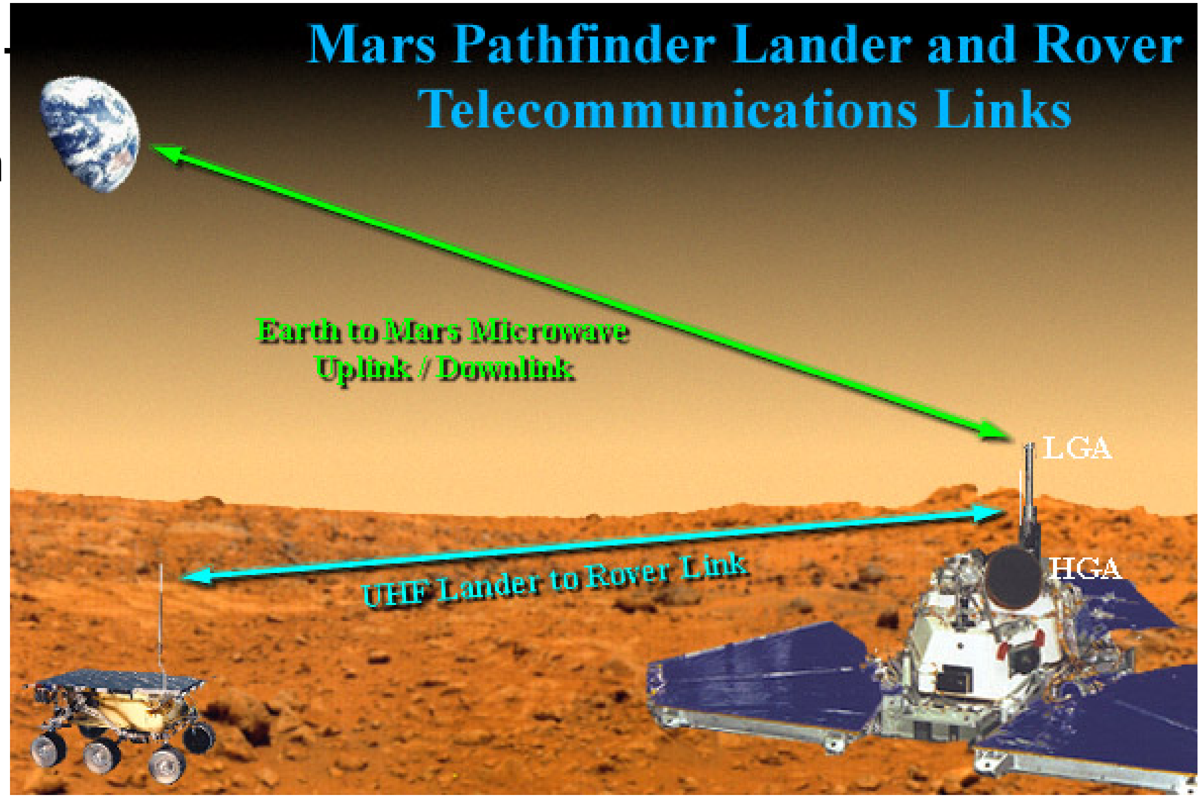
Motivation

• The first robot rover exploration of Mars was in 1997. A rover, named Sojourner send and receive information to the earth over the lander.

• Certain metallic or rocky structures and ground reflections near the antenna will distort its radiation pattern and cause holes or null zones to form.

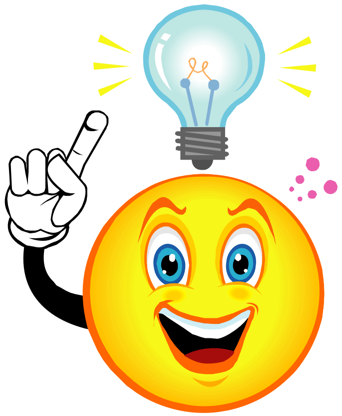
• This may caused poor signal reception.

• The other problem is that the rover can only send and receive information in 10m range.



Motivation

- A solution could be to send with a higher signal power to have a longer radio link.
- But power on mars is luxury!!! Otherwise, the problem of reflexions in not longer solve.
- We can use a multi rover system!!! If we use more rovers, it is possible to greatly enhance the strength of the signal transmitted in a given direction.



Motivation

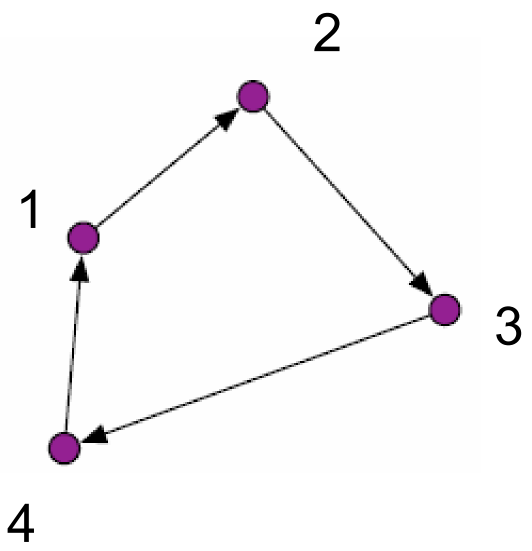
- When it’s time to communicate with the lander, the rovers arrange themselves in a suitable formation to become an antenna array in order to optimize the signal strength.

- The questions, we can are now :
  - How can we get a group of robot rovers, placed initially at random, to form a circle or other shape?
  - When is the convergence to a general geometric pattern reachable?

Cyclic pursuit / agreement problem

- We consider n > 1 robots with the kinematic model: $\dot{z}_i = u_i.$
- Whereby the robots can be model as points in the complex plane: $z_1, \dots, z_n \in C.$
- Let robot i be able to measure the position of robot i+1 (mod n).
- Then the output of our system is:

$$y_i = z_{i+1} - z_i, i = 1, \dots, n-1$$
$$y_n = z_1 - z_n.$$

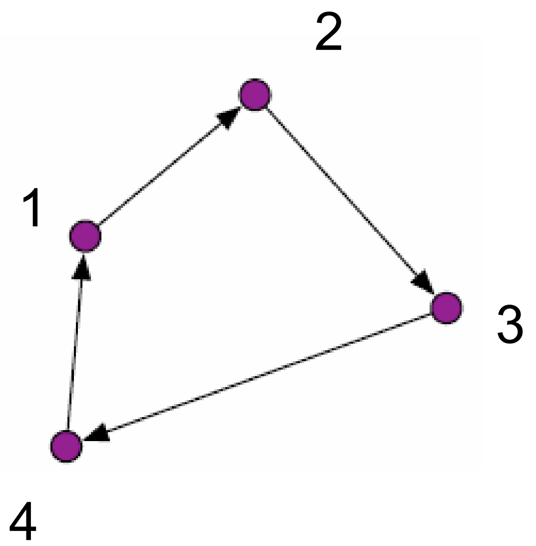


- The centroid of the $(z_1, \dots z_n)$ is defined as:

$$z_0 = \left(\frac{1}{n}\right)\sum_{i=1}^{n} z_i$$

Cyclic pursuit / agreement problem

- The control law is to steer over toward the next robot (closed loop controller):



$$\dot{z}_i = u_i,\ i = 1,\ldots,n$$

$$u_i = F_i y_i, F_i = 1$$

$$Then\ follows: \dot{z}_i = z_{i+1} - z_i$$

$$E.g.:\ for\ \ n = 4$$

$$\begin{cases} \dot{z}_1 = z_2 - z_1 \\ \dot{z}_2 = z_3 - z_2 \\ \dot{z}_3 = z_4 - z_3 \\ \dot{z}_4 = z_1 - z_4 \end{cases} \qquad \dot{z} = Mz, with\ \ M = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

Cyclic pursuit / agreement problem

- The Matrix M has one Eigenvalue at the origin since the row sum of M is 0.

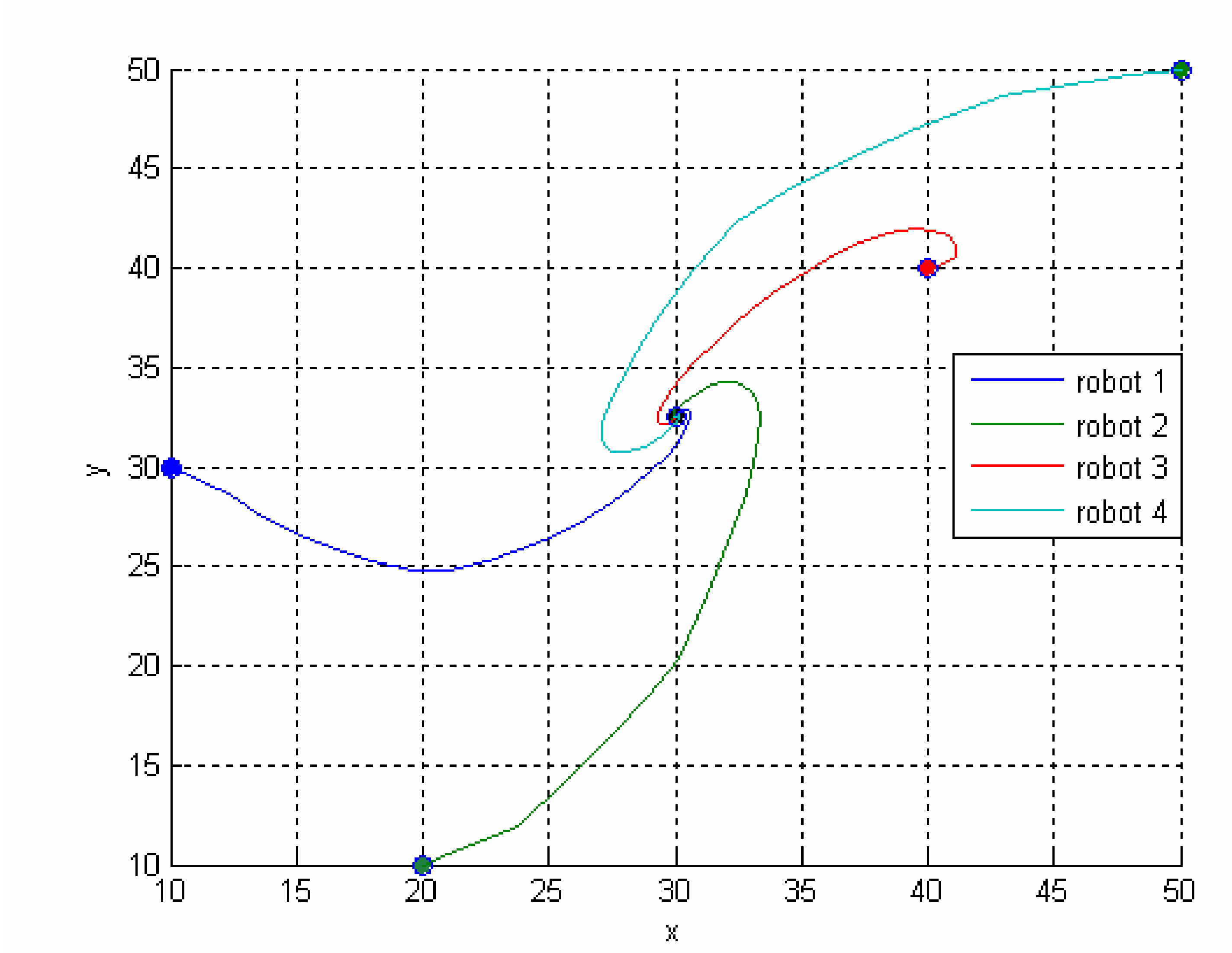
- All the other Eigenvalues of M have negativ real parts.

- Then it follows that for all initial locations of the agents, the centroid of the points is stationary and every z_i converge to this centroid. (agreement problem).

- We can achieve convergence to a point.

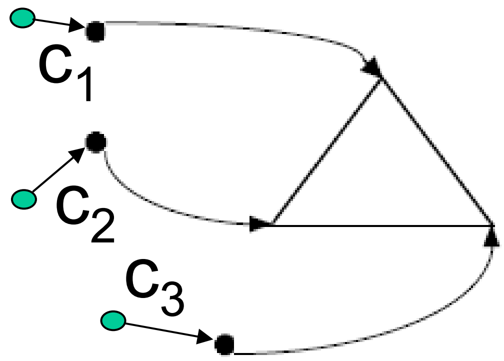
Cyclic pursuit / agreement problem

- The following simulations illustrate the behavior for 4 robots:



Cyclic pursuit / achievable formations

- Now instead of converge to a point, we can get three robots to form a triangle or more generally for a group of robot to achieve a general pattern, by a distributed stategy.



- Our control law is modified with the virtual displacement c, then follow:

$$\dot{z}_i = (z_{i+1} + c_i) - z_i, \qquad i = 1,\dots,n-1,$$
$$\dot{z}_n = z_1 + c_n - z_n$$
$$\dot{z} = Mz + c$$

Cyclic pursuit / achievable formations

- Let us assume that, d is the unique vector satisfying: $Md + c = 0.$

  Then follow with $\dot{z} = Mz + c$: $\dot{z} = M(z - d)$

- We have reduced our problem to the previous case and can then formulate the following theorem:

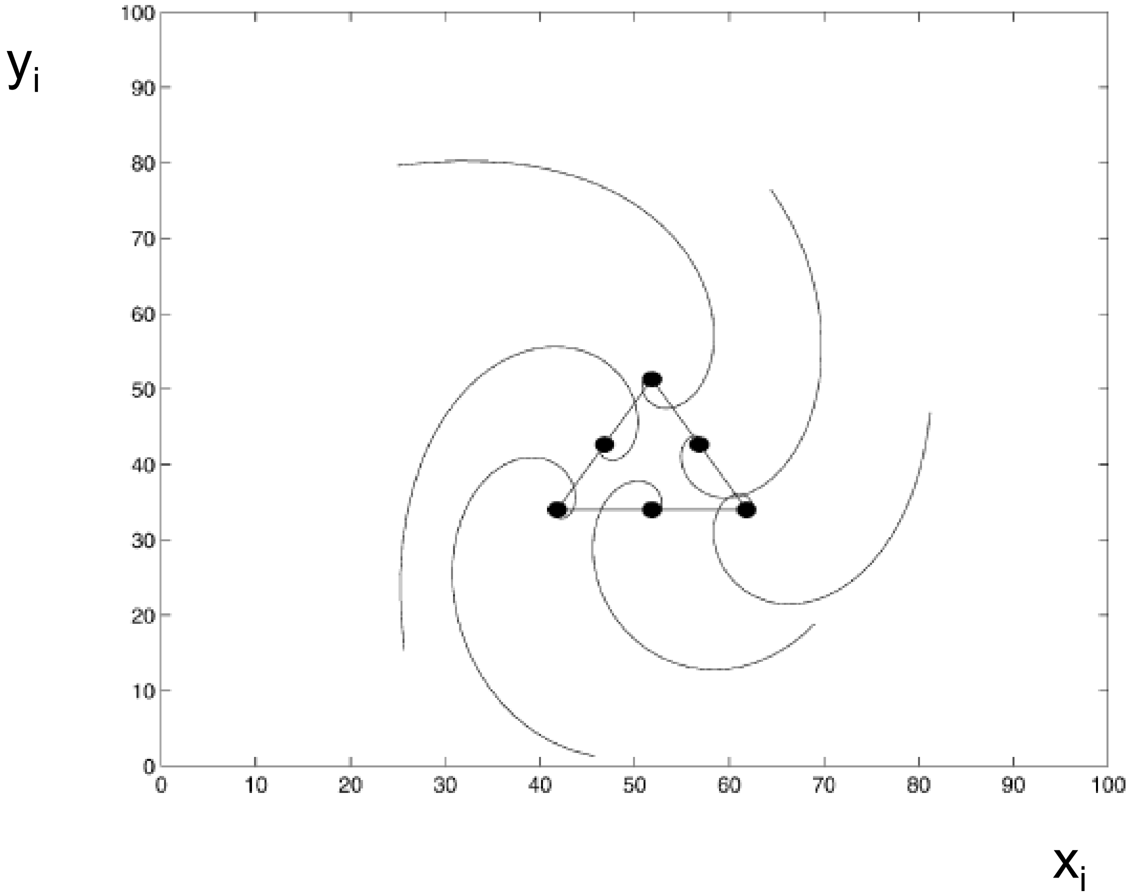
<u>Theorem:</u>

Concerning the previous system, assume the centroid of the points $c_1,\ldots,c_n$ is at the origin. Let d denote the unique vector satisfying $Md + c = 0.$ Then for every initial positions of the robots, the centroid of the points $z_1,\ldots z_n$ is stationary and robot i converges to this centroid displaced by d_i.

Cyclic pursuit / results

- The next simulation shows, how to achieve an equilateral triangle formation for six mobile agents with the virtual displacement c.

$$c = \left(-5 + j\sqrt{3}, -5 + j\sqrt{3}, 10, 10, -5 - j\sqrt{3}, -5 - j\sqrt{3}\right)$$

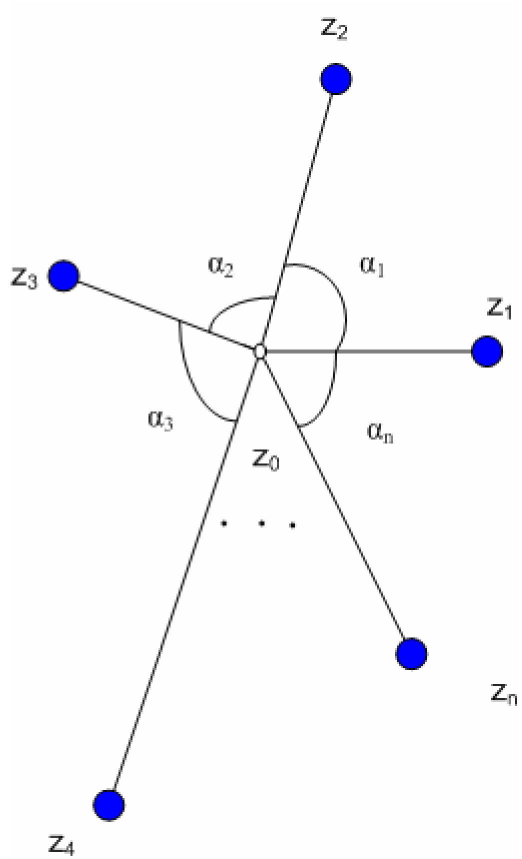


Cyclic pursuit / Collision avoiding

- Let us now consider the collision problem. Consider the n distinct points $z_1, \dots, z_n$ of the previous example, not all colinear. Let z_0 be their centroid and r_i be the distance between z_i and the centroid.

<u>Definition</u>: n points are said to be arranged in a counterclockwise star formation

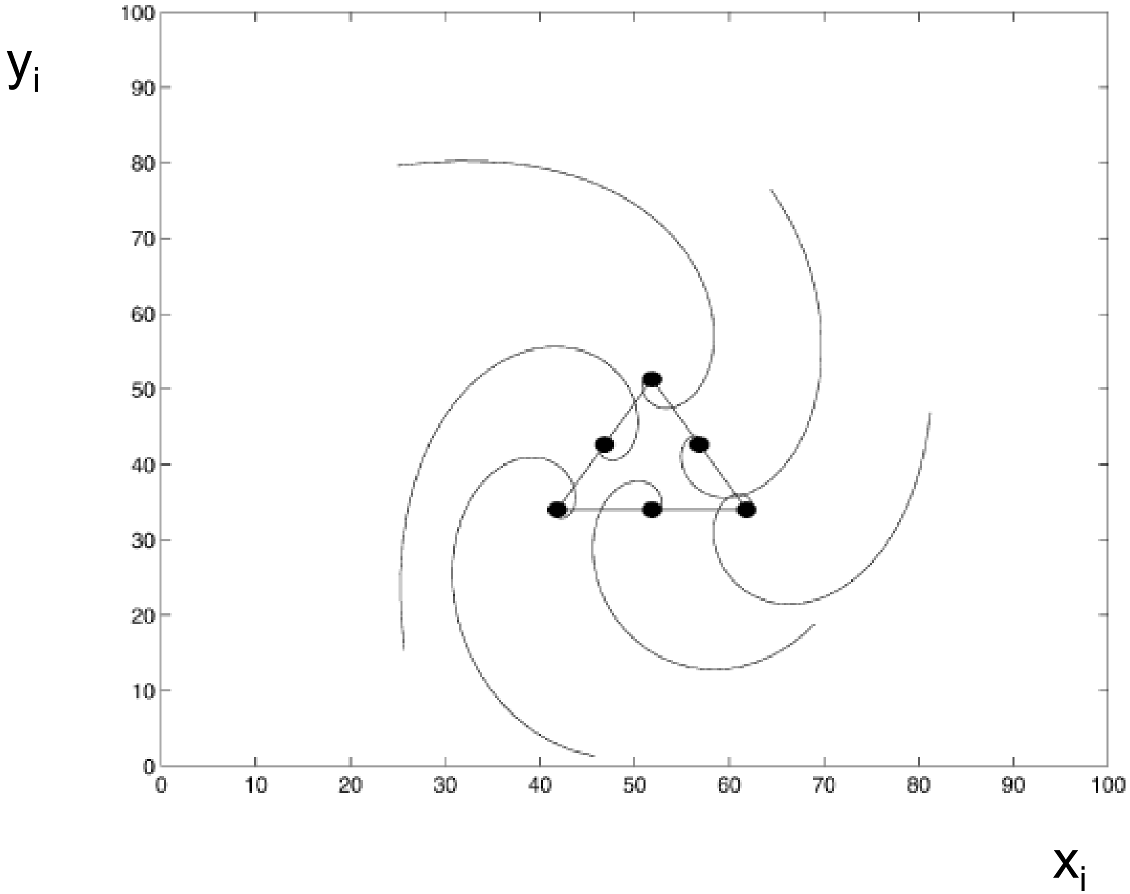
if $r_i > 0$ and $\alpha_i > 0$ for all $i = 1, \dots, n$ and $\sum_{i=1}^{n} \alpha_i = 2\pi$.



Cyclic pursuit / Collision avoiding

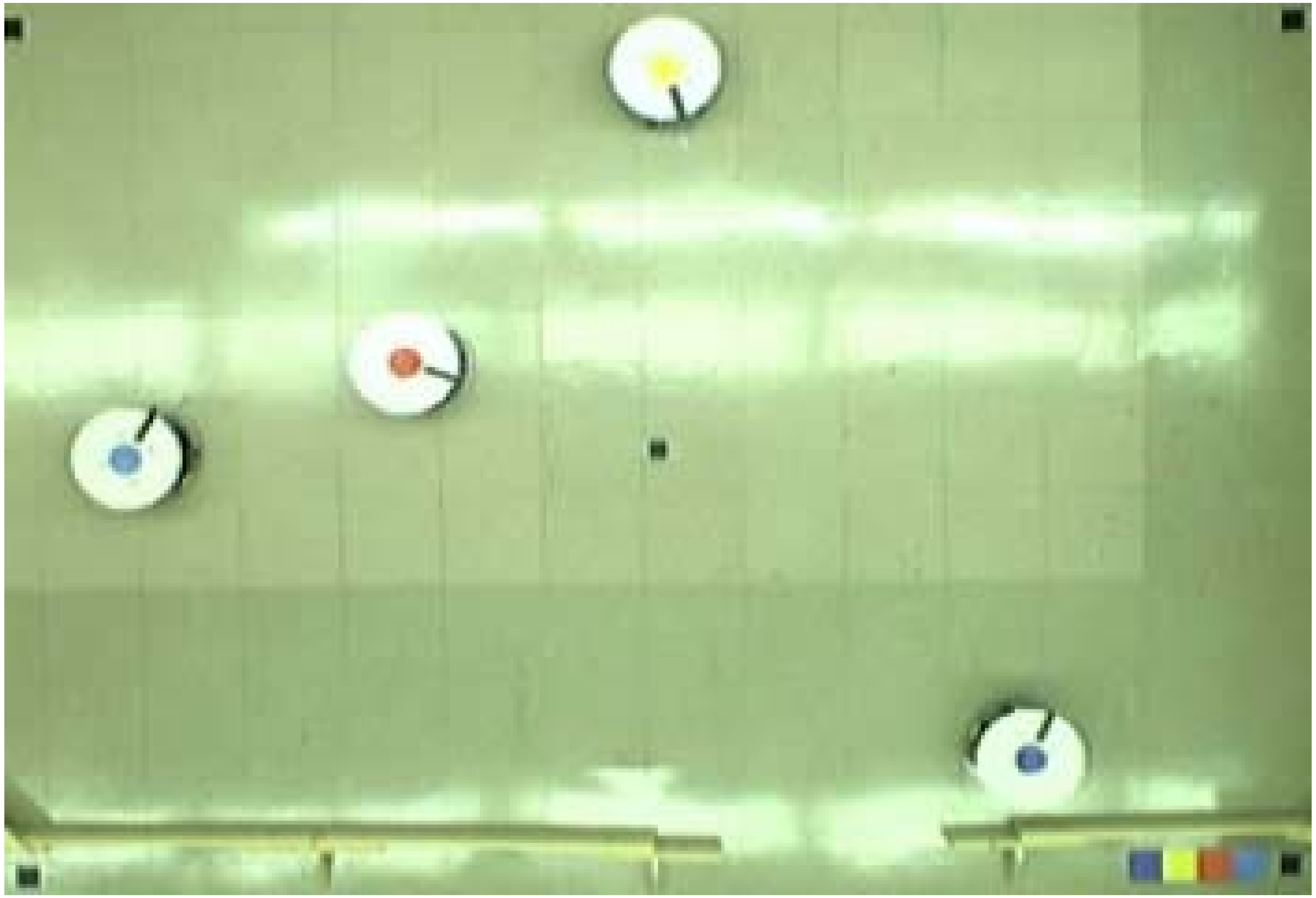
Theorem : Suppose n > 2 distinct points initially are arranged in a counterclockwise star formation. Under cyclic pursuit they remain in a counterclockwise star formation. (In particular, they never collide).

Simulation for 6 robots:



Cyclic pursuit / Demo

- The following video shows a realization of alignment using cyclic pursuit. For robot start randomly placed in a room und and self organise at constant speed,without supervisor intenvention, into polygon formation. Finally, a supervisor takes control of one of the robots (using a joystick) and leads the other through a door in the room.



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*http//www.control.utoronto.ca

Stabilization to a point / Unicycle

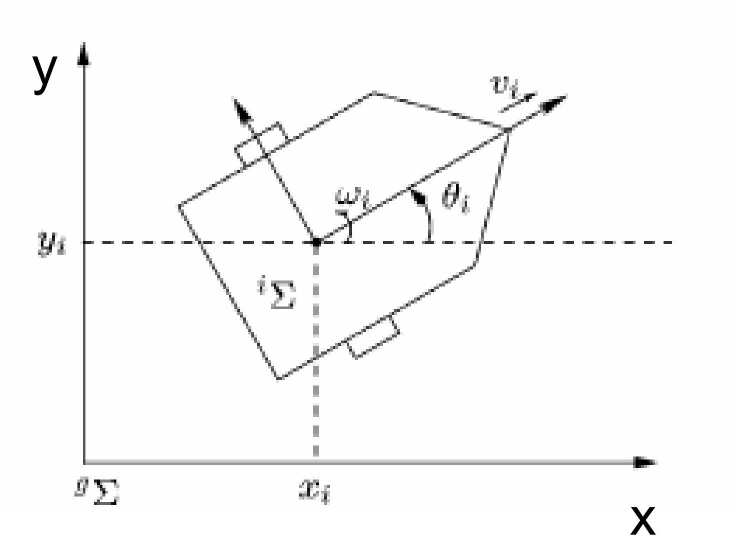
- The robots, we have considered until now were point-mass robots and were not really realistic sence the robots could move in all the directions (which is not really possible for real wheeled robots for example).
- The unicycle model consider this specificity and provide us a model that take in count this limitation.

Stabilization to a point / Unicycle - Model description

- For this task, let us consider the n wheeled vehicles with coordinates (x_i, y_i, θ_i), i=1,…,n.

  The location of the vehicle in the plane is $z_i = [x_i \; y_i]^T$ and each vehicle has limited field of view.

  Each vehicle is described by:



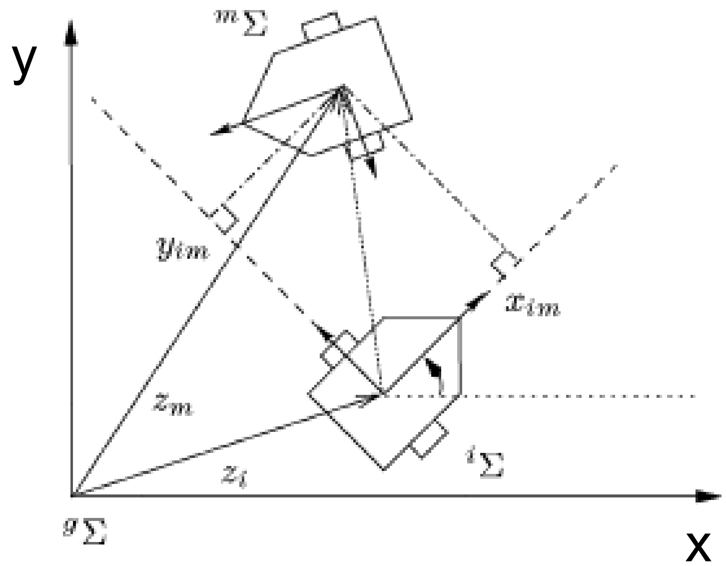
$$\begin{cases} \dot{x}_i = v_i \cos(\theta_\mathrm{i}) \\ \dot{y}_i = v_i \sin(\theta_\mathrm{i}) \\ \dot{\theta}_\mathrm{i} = \omega_i \end{cases} \quad or \quad \begin{cases} \dot{z}_i = v_i e^{j\theta} \\ \dot{\theta}_\mathrm{i} = \omega_i \end{cases}$$

  (where v_i is the velocity of the vehicle).

Stabilization to a point / Unicycle - Model description

- We construct a moving frame ${}^{i}\sum$, the Frenet-Serret frame, that is fixed on the vehicle.
- Let r_i be the unit vector tangent to the trajectory at the current location of the vehicle and let s_i be r_i rotated by $\pi / 2$.
- Thus, $\dot{z}_i = v_i r_i$, since the vehicle is moving at the speed v_i.
- Since vehicle i can only get the relative positions of a subgroup N_i of vehicles with respect to its own Frenet-Serret frame:

$$\begin{cases} x_{im} = (z_m - z_i) \cdot r_i \\ y_{im} = (z_m - z_i) \cdot s_i \end{cases} \quad m \in N_i$$



Stabilization to a point / Local information controller- Problem Statement

- Let us now define a local information controller:

  A controller (v_i,w_i), i=1,…,n, is said to be a local information controller if:

$$\begin{cases} v_i = g_i(t, x_{im}, y_{im}) \,|\, m \in N_i \\ w_i = h_i(t, x_{im}, y_{im}) \,|\, m \in N_i \end{cases}, \quad i = 1,\dots,n$$

$$\text{where } g_i \text{ is such that } \{(\forall m \in N_i)\; z_m = z_i\} \Rightarrow \{v_i = 0\}$$

- Now we come to our main task, namly to find if possible a local information controller such that for all $(x_i(t_0), y_i(t_0), \theta_i(t_0)) \in R^3$, i = 1,…,n and for all t_0:

$$(\exists z_{ss} \in R^2)\; (\forall i)\; \lim_{t\to\infty} z_i(t) = z_{ss}.$$

- The directed graph defined by the information flow plays a key role. We need a review of some basic notions in graph theory.

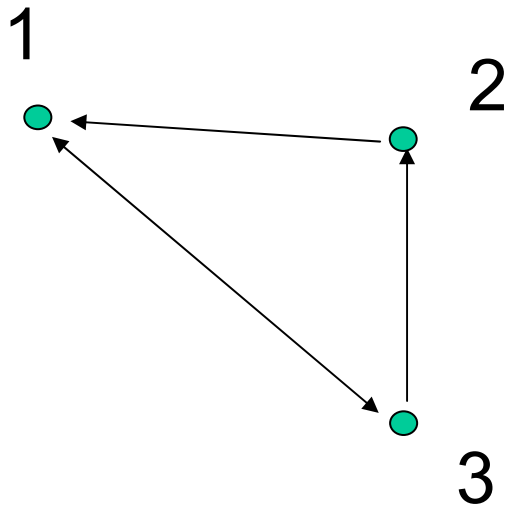
Background about graph theory

- A graph G = (V,E) is defined to be a pair of sets V and E.

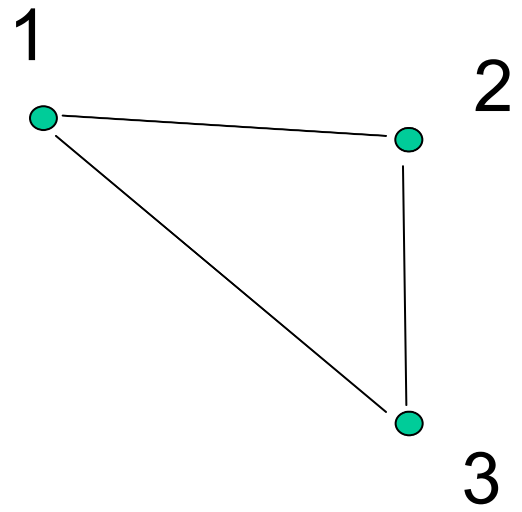
  V is the set of a limited number of nodes and E is the set of arcs.

- There are two important types of graphs: the directed

  graph(digraph) and the indirected graph.



Directed graph



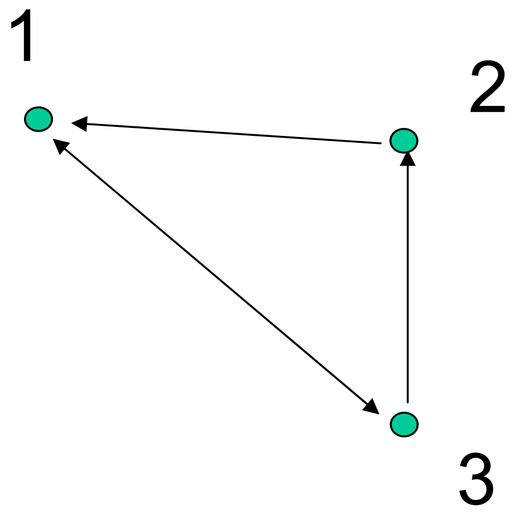
Indirected graph

Background about graph theory

Several matrix can be associated with a directed graph:

- The adjacency matrix $A = (a_{ij})$: If there is an arc from node i to node j then the entry a_{ij} equals 1, otherwise 0.
- The degree matrix $D = (d_{ij})$ is defined to be a n×n diagonal matrix where the diagonal entry d_{ii} is the out-degree of node i.
- Finally the graph Laplacian is the matrix defined as $L=D-A$.

  E.g.:

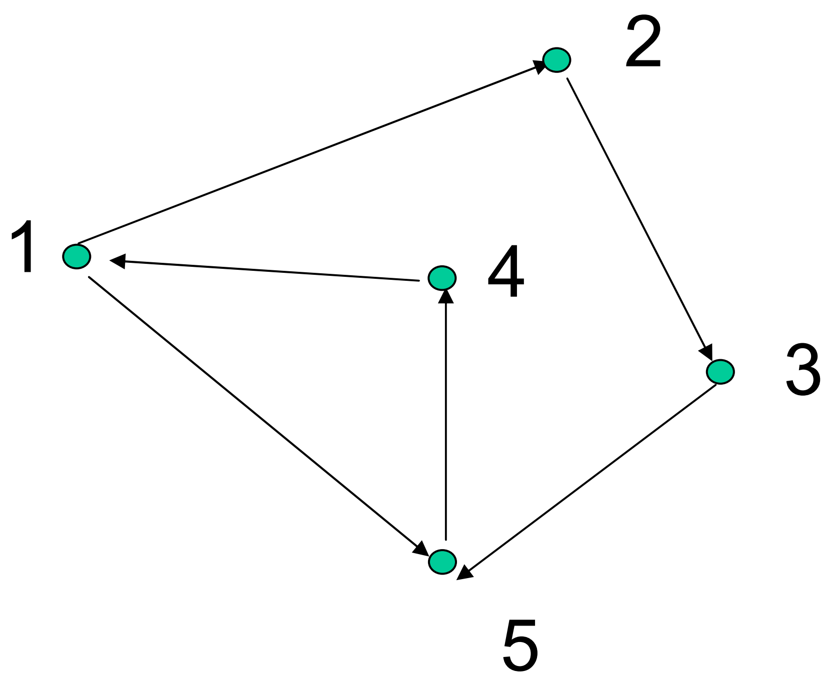


$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = D - A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

Background about graph theory

- If there is a path in G from one node i to another node j, then j is said to be reachable from i written. If not,j is said to be not reachable from i.
- If a node is reachable from every other node in G, then we say it is globally reachable.
- E.g.:



- a sensor digraph (directed graph) models the information flow, where a link from node i to node j indicates that vehicle i can sense the position of vehicle j.

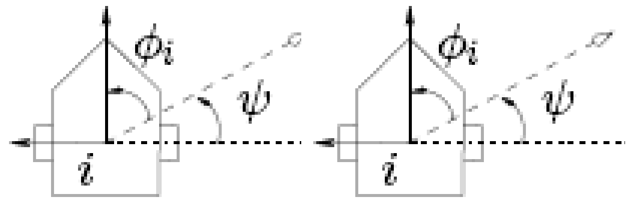
Stabilization to a point / Unicycle

- Now we come back to the unicycle.
- The information flow is central since the vehicle are coupled through the sensor information flow.
- we assume the information flow to be static.
- It can be proven that, the problem of convergence can be solved if and only if the graph has a globally reachable node. An algebraic characterization of this property, is that, the Laplacian L of the graph has a simple eigenvalue 0.
- The information flow produce a kind of symmetry in the system equations with respect to the x and y coordinates.
- Moreover, we can find a local controller, so that the convergence is achieved.

Stabilization to a point / Unicycle

- The idea is that each robot each permanently turn around his own center self and at the same time to follow the centroid of his Neighbor like in cycle pursuit.
- we then use the following control law:

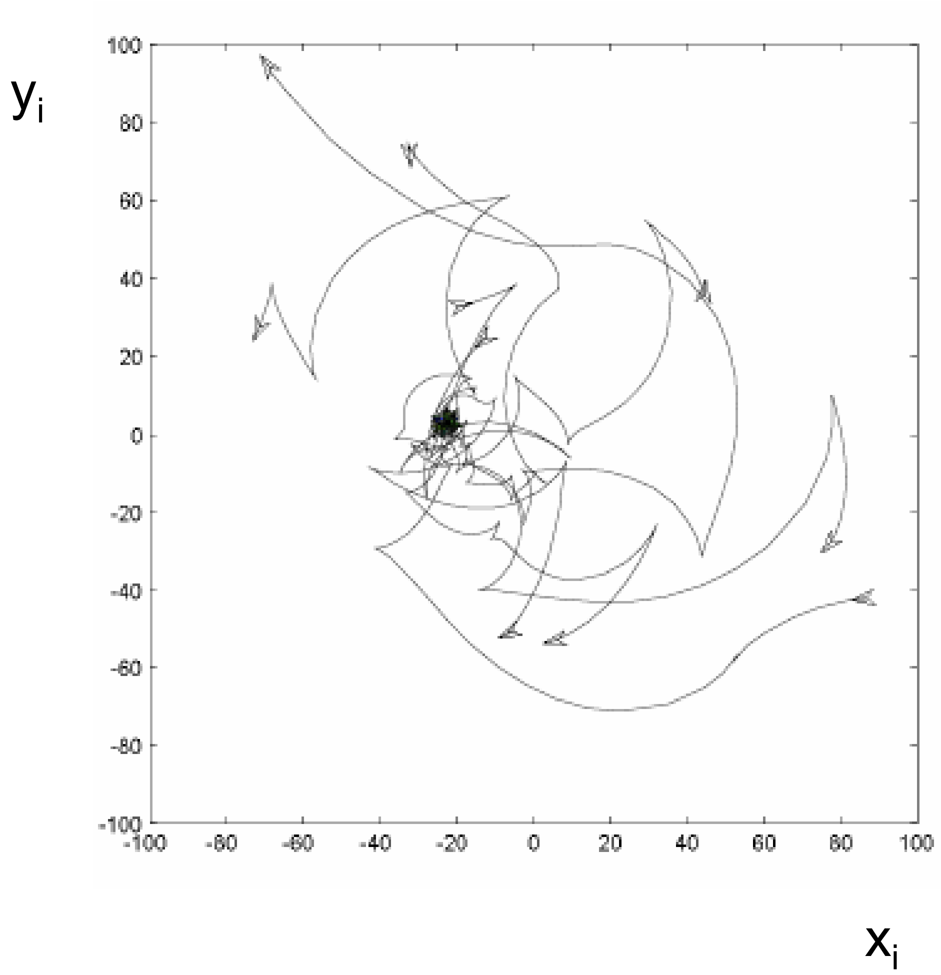
$$\begin{cases} v_i(t) = k \sum\limits_{m \in N_i} x_{im} & k > 0 \\ \omega_i(t) = \cos(t) \end{cases}$$



- Using this controller, we can prove that, the convergence to a point can be achieved.

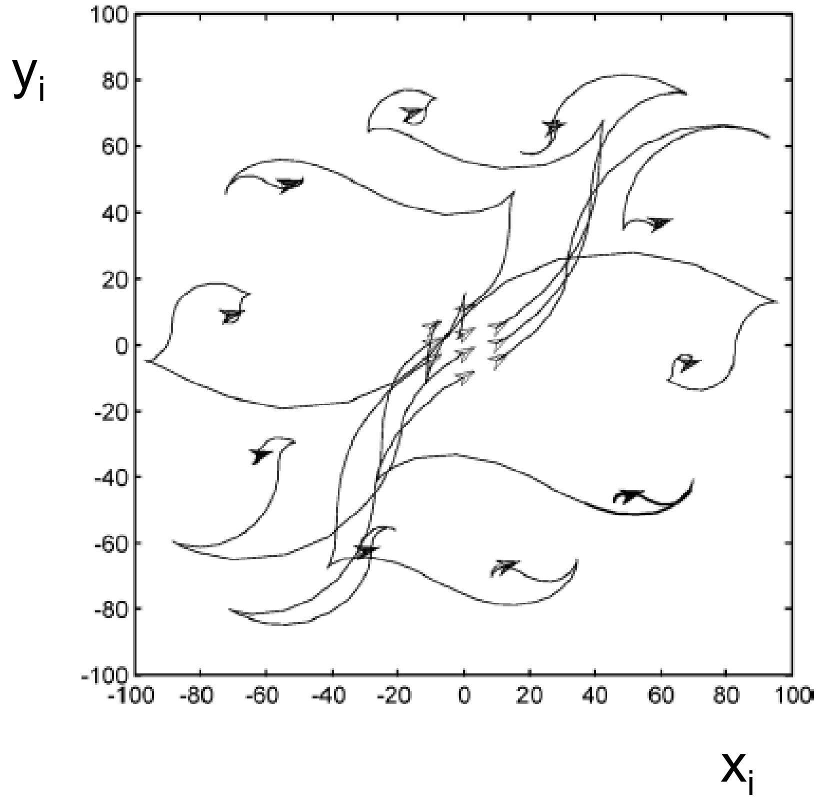
Stabilization to a point / Unicycle

The following simulation shows the convergence to a point:



Summary and perspectives

- The feasibility of formation for cyclic pursuit and for the more realistic case namly unicycle were studied.
- Note that convergence to a more general geometric pattern can also be achieve and a control law can be found.
- The idea like by the convergence to a point.
- The following simulation shows the example of cycle formation:
- For further information see refences



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