# The tube wave reflection from borehole fracture

***Andrey Ponomarenko***
StPSU, Faculty of physics
Laboratory for Elastic Media Dynamics

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**Abstract**

This work is about the wave transmission in the tube which situated in the homogeneous elastic space (fig.1). The tube is filled with non-viscous water and it intersects with the quite thin fracture. What does it mean "thin" I'll say later. In the tube or borehole the wave can exists. It is called Stoneley wave. In particular I focused on the reflection phenomena arising when Stoneley wave transmits in the tube and encounters the fracture intersecting borehole. Part of wave energy is reflected, part passes the fracture and another part transforms into a mode that propagates outward into the fracture (fig.1). When you have real borehole for the oil production, the geological formation in which you have made

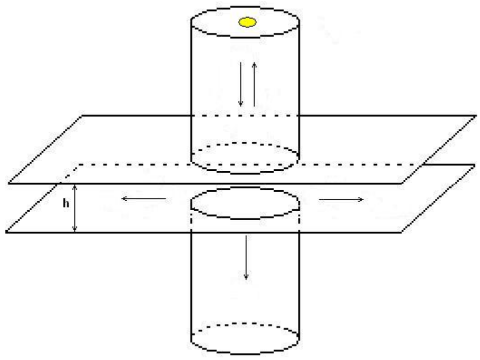


Figure 1: The model

the borehole can have fractures in it. And such fractures because of their duration in the formation can be used for transporting oil from the place where it is to the borehole to be pumped out. So such parameter as the effective fracture width is important to be known or at least to be estimated to have more information about the productivity of the borehole.

Sometimes another situation occurs: there are predictions that the formation is so layered, that fracture existence is possible. For example, may be some time ago there were borehole with fractures and you produced oil with the help of them well, but they have closed in the power of some geological reasons. And you want to continue the business. So you pump in the fracture the water under the high pressure and after it fracture appears again. And again in this case it is useful to have information about fracture dimensions for estimating productivity and for other technical reasons. How can we get it? With the help of reflection coefficient of the tube wave! Imagine the source of the acoustic wave (fig.1) in the borehole. This source sends the signal with not very high amplitude (if the amplitude will be quite high, it is the possibility then appears to break the borehole). If there are any fractures you will receive the reflected signal. From this signal it is possible to get information about the effective width of the fracture. And, of course, if you have the analytical model of described situation you would facilitate enough the determination of the effective fracture width.

I consider the analytical model which can interpret the tube wave reflections from fractures, then I calculate formulae for the reflection and transmission coefficients of the tube wave. Finally, I compare the results of analytical calculations with the results of the finite-difference (FD) modeling.

## The model

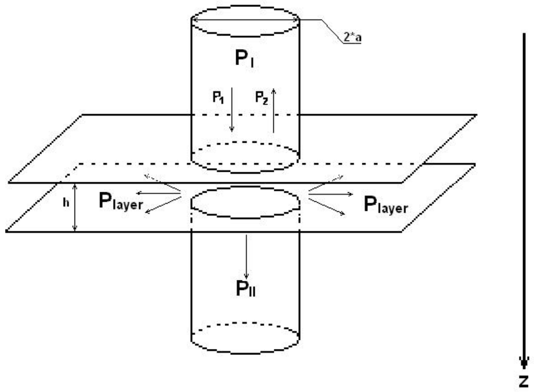


Figure 2: The model

There is the same picture (fig.2). But all waves are denoted by the letter P because they are pressure waves. $P_1$ is incident wave, $P_2$ is reflected wave, $P_{layer}$ is guided wave in the fracture, $P_{II}$ is transmitted wave. I assume that the fracture is perpendicular to the borehole axis and parameters a - the borehole radius and h - the effective fracture width are both smaller than the characteristic wavelength in the tube: $a << \lambda_{TW} = \frac{V_{TW}}{\nu}$, $h << \lambda_{TW} = \frac{V_{TW}}{\nu}$. So the frequency $\nu$ should be low enough. I will consider range of frequencies from 0 to 2000 Hz. The Stoneley wave in the tube can be approximated by a tube wave with uniform pressure distribution across the borehole (fig.3) at low frequencies and on some distances from the source.

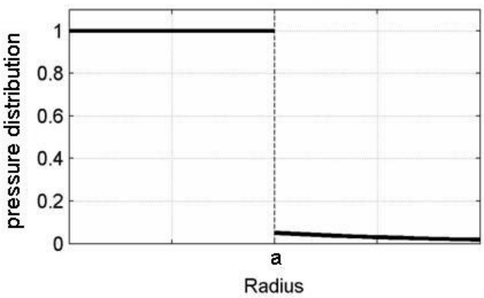


Figure 3: Pressure distribution in the tube

I introduce coordinate axes r and z with the fracture location at z=0 then waves can be expressed by formulae. I assume that the source signal is harmonic and can be presented in the form of Fourier integral (it should be real, because the signal is real):

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(\omega)e^{i\omega t}d\omega = \Re\left(\frac{1}{\pi}\int_{0}^{+\infty} f(\omega)e^{i\omega t}d\omega\right). \tag{1}$$

Then the wave fields in the frequency domain are defined by:

$$P_I = P_1 + P_2 = e^{-ik_T z} + R(\omega)e^{ik_T z} \tag{2}$$

$$P_{II} = T(\omega)e^{-ik_T z} \tag{3}$$

$$P_{layer} = Q(\omega)H_0^2(k_r r), \tag{4}$$

where $R(\omega)$ is reflection coefficient of the tube wave, $T(\omega)$ is transmission coefficient of the tube wave, $Q(\omega)$ is so called guided wave's coefficient. The wave numbers of the tube wave $k_T = \frac{\omega}{V_{TW}}$ and the wave number of the guided wave is $k_r = \frac{\omega}{V_{GW}}$. Wave numbers are expressed as cyclic frequency divided by velocities. Velocity of the tube wave depends on the elastic parameters of the fluid in the tube and surrounding elastic medium and can be obtained for the low frequency domain by:

$$V_{TW} = \left( \frac{1}{V_f^2} + \frac{\rho_f}{\rho_s V_s^2} \right)^{-\frac{1}{2}}, \tag{5}$$

where $V_f$ is velocity of the longitudinal wave in the fluid filled the tube, $\rho_f$ is density of this fluid, $\rho_s$ is density of the elastic space, $V_s$ is transverse wave velocity in the elastic space.

The pressure in the fracture is the pressure of such called guided wave and it is expressed through Hankel function.

## Symmetrical guided wave in the fracture

At first notice that guided wave is axial symmetrical relative to the borehole cylinder's axis. We solve the wave equation:

$$\Delta p = \frac{1}{V_{GW}^2} \frac{\partial^2 p}{\partial t^2}. \tag{6}$$

With the help of Fourier time transformation (1) turn our equation to the spectrum domain. Now it is the Helmholtz equation:

$$\Delta P + \frac{\omega^2}{V_{GW}^2} P = 0. \tag{7}$$

We should solve it in the cylindrical coordinates, which in our consideration are polar so the Laplacian operator will have only two parts:

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2} \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}. \tag{8}$$

Then (7) becomes the Bessel equation of zero order (9). It has solutions of Bessel and Neiman functions of the zero order.

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} P + \frac{\omega^2}{V_{GW}^2} P = 0 \tag{9}$$

$$P = C H_0^{(2)} \left( \frac{\omega}{V_{GW}} r \right) \overset{r \to +\infty}{\longrightarrow} C e^{-i \frac{\omega}{V_{GW}} r} \tag{10}$$

We choose their linear combination which is usually called Hankel function. Why have we chosen Hankel function of second type and not first?! Look at its asymptotic when r is infinite (10). According to previously chosen sign of the time Fourier (1), solution of the Bessel equation describes outgoing wave only if it has form of (10):

$$p(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} P(\omega) e^{i\omega t} d\omega \sim C e^{-i \frac{\omega}{V_{GW}} r} e^{i\omega t} \tag{11}$$

So we have the form of answer for the guided wave in the fracture (10). The magnitudes for velocity of this wave $V_{GW}$ can be obtained from the dispersion equation for the slab of fluid bounded on each side by a semi-infinite elastic medium.

## The equations for coefficients

It is necessary now to have the system of the equations to obtain frequency-dependent functions of coefficients. First equation is the equation of the pressure continuity at z=0 or at the whole volume of this small cylinder - see (2),(3),(4) and fig.4 and fig.5. The pressure is uniform in this region where fracture intersects the borehole because of the long-wavelength approximation which we have assumed. So now we have the first equation to obtain $R(\omega)$ and $T(\omega)$:

$$1 + R(\omega) = T(\omega) = Q(\omega) H_0^2(k_r a) \tag{12}$$

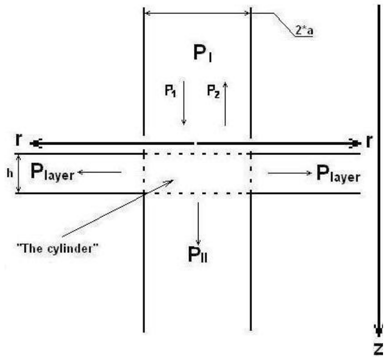


Figure 4: 2D model's cut

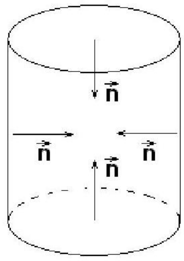

Figure 5: Cylinder

$Q(\omega)$ we express through $R(\omega)$ and $T(\omega)$, because (12) includes two equations inside itself.

The second equation is the equation of the continuity of fluid flux through the cylinder (fig.5):

$$\mathrm{div}\overrightarrow{V} = 0 \tag{13}$$

Integrate this equation by the volume of "cylinder", use Ostrogradskiy-Gauss theorem and obtain:

$$\int\limits_U \mathrm{div}\overrightarrow{V} dU = 0 = \int\limits_S \left(\overrightarrow{V}, \overrightarrow{n}_{ex}\right) dS = -\int\limits_S \left(\overrightarrow{V}, \overrightarrow{n}_{in}\right) dS, \tag{14}$$

where $U$ is the cylinders' volume and $S$ is the area of the cylinder's surface. I choose the internal normal vector for next calculations

$$\int\limits_S \left(\overrightarrow{V}, \overrightarrow{n}_{in}\right) dS = 0, \tag{15}$$

because the directions of this normal vector and the incident wave are the same (fig.4 and fig.5).

According to the choice of the normal, this vector equation (15) transforms to the scalar sum

$$\pi a^2 \left(V_I - V_{II}\right) - 2\pi a h V_{layer} = 0, \tag{16}$$

where $\pi a^2$ and $2\pi a h$ are areas of the cylinder base and lateral area, also there are absolute values of the wave velocities, and

$$V_I = |\overrightarrow{V}_I| = |\overrightarrow{V}_1 + \overrightarrow{V}_2|. \tag{17}$$

Now we have to connect wave pressures which we have in the initial equations (2),(3),(4) with the velocities from (16). I use the Euler equation for non-viscous ideal fluid:

$$\rho_f \frac{d\overrightarrow{\mathrm{v}}(t)}{dt} = -\overrightarrow{\mathrm{grad}}\ \mathrm{p}(t). \tag{18}$$

And if we express both pressure and velocity in the Fourier integral (1), we obtain:

$$\overrightarrow{V}(\omega) = -\frac{\overrightarrow{\text{grad}} P(\omega)}{i\omega\rho_f}. \tag{19}$$

Finally we have system of equations (16) and (12):

$$\begin{cases} \pi a^2 \left(V_I - V_{II}\right) - 2\pi a h V_{layer} = 0 \\ 1 + R(\omega) = T(\omega) = Q(\omega) H_0^2(k_r a) \end{cases} \tag{20}$$

The first equation here was obtained from the continuity of the flux, velocities can be obtained from the Euler equation (19) with the help of (17). The second equation is the equation of the pressure uniform. There are two equations and there are two unknowns. Third unknown Q can be expressed through R and T. After some algebra I obtained next expressions for coefficients:

$$R = \frac{\frac{ihk_r H_1^2(k_r a)}{a H_0^2(k_r a)}}{k_T - \frac{ihk_r H_1^2(k_r a)}{a H_0^2(k_r a)}} \tag{21}$$

$$T = \frac{k_T}{k_T - \frac{ihk_r H_1^2(k_r a)}{a H_0^2(k_r a)}} \tag{22}$$

$$Q = \frac{k_T}{(k_T - \frac{ihk_r H_1^2(k_r a)}{a H_0^2(k_r a)}) H_0^2(k_r a)} \tag{23}$$

They have frequency dependence through the wave numbers $k_T$ and $k_r$:

$$k_T = \frac{\omega}{V_{TW}}, \qquad k_r = \frac{\omega}{V_{GW}}.$$

And of course, the magnitudes of the tube radius and width of the fracture are the parameters. So the reflection coefficient depends on the parameter h, and we can estimate its value.

# The finite-difference model

And now I consider the finite difference modeling. I am going to describe a little finite difference code which we use in our laboratory. There is the special file (see some parts of it interface on the fig.6 and fig.7), where there is the scheme of the model (fig.6), place to fix all coordinates, types of media and their parameters (fig.7), grid information and some information about data output (you have to fix coordinates of the receivers and components of the fields). Also there is place there you have to fix type of the source.

It is enough for homogeneous nonporous medium to set the density ("rof"$=\rho_f$ or "ros"$=\rho_s$), bulk modulus ("akf"$=\lambda$ or "aks"$=K$) and shear ("amus"$=\mu$) modulus. What is the bulk modulus? It describes relative modification of the little volume in the media. For non-viscous fluid it is

$$\lambda = \rho_f V_f^2,$$

and for elastic medium it is expressed by:

$$K = \lambda - \frac{4}{3}\rho_s V_s^2 = \lambda - \frac{4}{3}\mu.$$

In the input file of the code each modulus is expressed through the density values and the values of longitudinal and transverse wave, they are called p (primarily) and s (shear, secondary) waves. I show their magnitudes in the table below.

Then the grid information is set in the input file. And the very important thing is the inequality for grid steps, which denoted as "hrmax" and "hzmax" according to the each axis:

$$\text{hrmax} << \frac{\lambda_{\min}}{14}, \quad \text{hzmax} << \frac{\lambda_{\min}}{14},$$

where $\lambda_{min}$ is the minimum wavelength of all waves which can exist in the model. If inequality doesn't true, the results of the computation cannot be good.

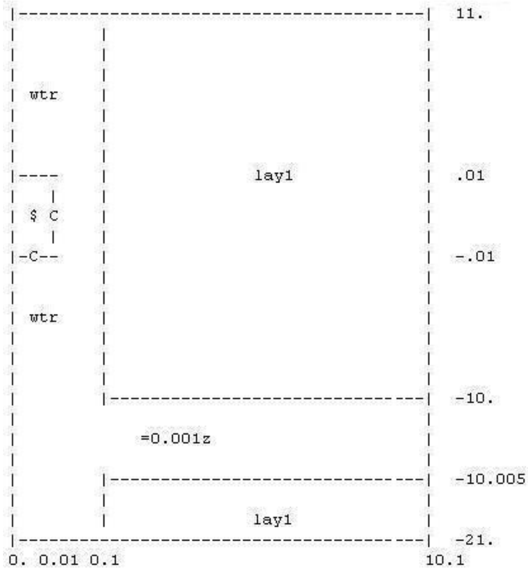


Figure 6: The scheme of the model in the input file

list of media

| media | rof | g0r | g0z | eta | akf | ros | aks | amus |
|---|---|---|---|---|---|---|---|---|
| wtr | 1.e3 | -0. | -0. | -0. | 2.25E+09 | -0. | -0. | -0. |
| lay1 | -0 | -0. | -0. | -0. | -0. | 2.5E+03 | 3.25E+10 | 2.25E+10 |

mudcake membrabe stiffness list
a=0

Figure 7: Media parameters

| medium | Elastic (lay1) | Fluid (wtr) |
|---|---|---|
| Longitudinal velocity (m/s) | 5000 | 1500 |
| Shear velocity (m/s) | 3000 | - |
| Density (kg/$m^3$) | 2500 | 1000 |

There are some other parameters such as maximum time for calculations and etc. But I think it is no need to talk about them now and any time you need to use them you can open and use special manual of this code [2].

## The FD seismogram

Computational result of this code can be shown with the help of some special program as the seismogram (fig.8). The horizontal axis is the axis of receivers. They are situated in the fracture. Vertical axis is the time axis. Fracture is situated on the depth of 10 m. and the distance from the source is also 10 m. So z-coordinate of the source is 0 m. The radius of the tube equals 0.1 m and fracture width equals 0.005 m. Incident, reflected and transmitted waves easy to be seen on the seismogram. For the better fields visualization there is the possibility to change the maximum amplitude which you want to observe on the

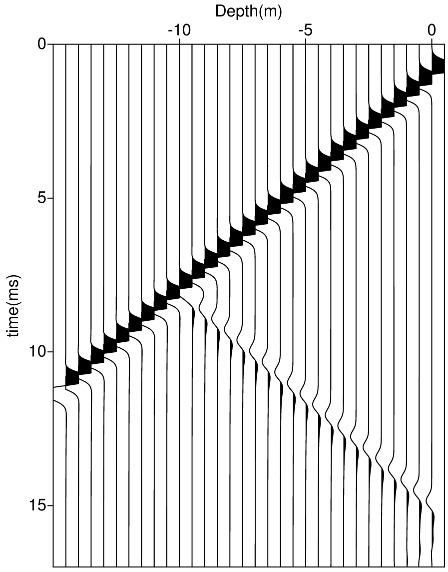


Figure 8: Finite-difference seismogram

seismogram.

## Comparison of the FD modeling results with analytic approach

One of the tasks for the work was the comparison of the analytical results with the finite-difference ones. Analytical results are the reflection and transmission coefficients (21) and (22), which were obtained above. The analytical seismogram (red) was obtained with use of these coefficients and it is very close to the finite-difference one (black) - fig.9 and fig.10. Also I made the comparison of the absolute value

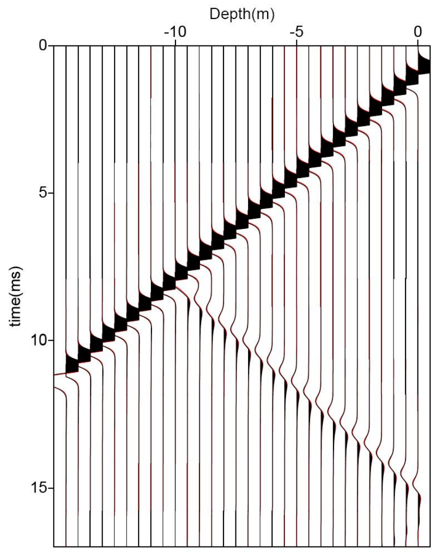


Figure 9: FD (black) and analytical (red) seismograms

of reflection coefficient (fig.11). The absolute value of reflection coefficient was obtained from the finite-difference seismogram by dividing the spectrum of the reflection signal to the spectrum of the incident signal. And there is the excellent agreement between absolute values of reflection coefficients too (fig.11). So I verified my analytical calculations for reflection and transmission coefficients of the tube wave in the

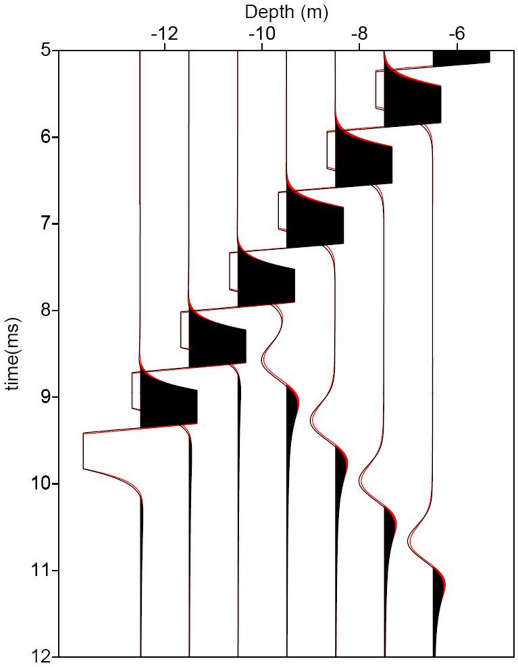


Figure 10: Enlarging of the previous figure

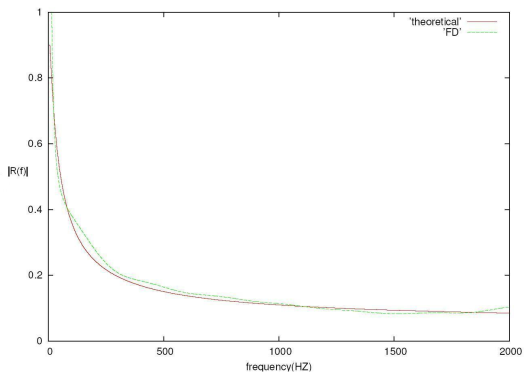


Figure 11: Absolute value of reflection coefficient, h=0.005 m.

long-wavelength approximation. As it was mentioned above, this result can be useful for estimating of the well productivity and for introducing better understanding of the interaction process between the tube wave and the fracture.

Another task was to compare absolute values of reflection coefficients calculated analytically and by finite-difference code for the different models where the width of the fracture changes. I made such comparison for several models. Medium parameters are set in the table below:

| medium | Elastic (lay1) | Fluid (wtr) |
|---|---|---|
| Longitudinal velocity (m/s) | 4200 | 1500 |
| Shear velocity (m/s) | 2500 | - |
| Density (kg/$m^3$) | 2700 | 1000 |

h=0,05 m. (fig.12), h=0,2 m. (fig.13), h=0,4 m. (fig.14), h=3,0 m. (fig.15). In spite of the fact that conditions of long wavelength approximation $h << \lambda_{TW} = \frac{V_{TW}}{\nu}$ not satisfied for some models at some frequencies (frequency range is 0-2000 Hz, peak frequency is 1000 Hz, $V_{TW} \sim 1400$m/s), results are good - they show good agreement between analytical calculations and finite-difference ones. Only the last fig.15 has the biggest discrepancy of results in compare with all showed pictures; it is predictable. So such result of verifying the application of analytic formulae for the wide range of models with different range of fractures can be useful in the estimating of the well-fracture's system parameters. And time of analytical calculations too smaller than the time of finite-difference code's similar calculations.

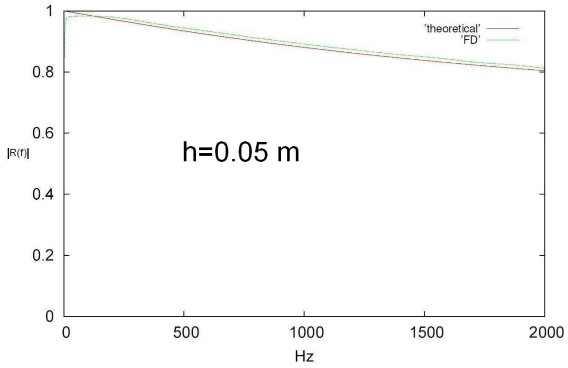


Figure 12: Absolute value of reflection coefficient, h=0.05 m.

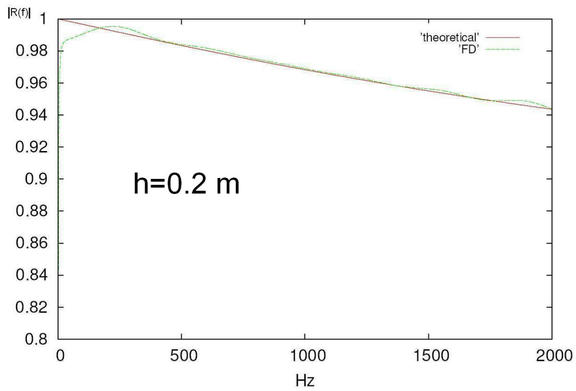


Figure 13: Absolute value of reflection coefficient, h=0.2 m.

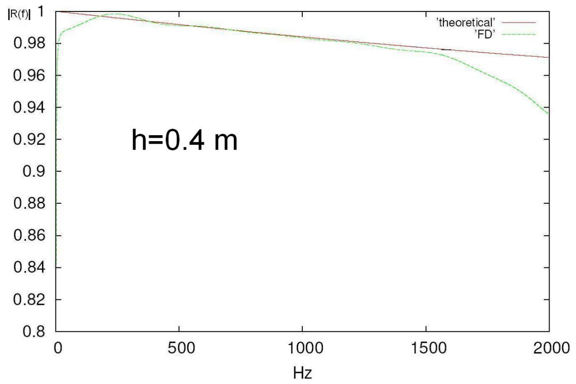


Figure 14: Absolute value of reflection coefficient, h=0.4 m.

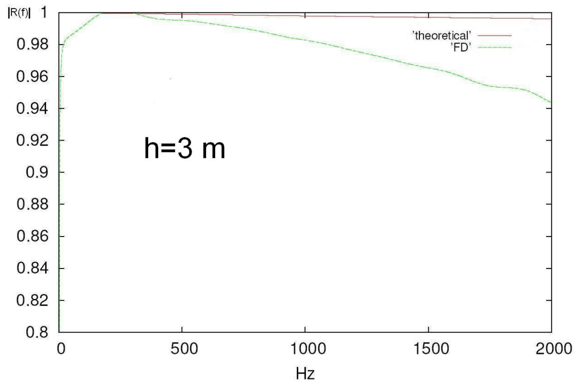


Figure 15: Absolute value of reflection coefficient, h=3.0 m.

# Conclusions

- The analytical formulae of the wave coefficients were derived. They showed excellent agreement with finite-difference modeling
- We can obtain better physical insight into the interaction of the tube wave with the fracture
- We can use analytical formulae for the cases with wide range of models
- It is possible to use obtained formulae for the estimating of the well-fracture system's parameters and, consequently, the estimating of the well productivity. It requests too less time than the time of finite-difference code's similar calculations.

# Acknowledgements

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# References

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