Automatic Discovering of Geometric Theorem by Computing Gröbner Bases with Parameters

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Abstract

A geometric statement of equality-type consists of two parts: hypothesis and conclusion. Both hypothesis and conclusion can be expressed in terms of polynomial equations in a number of free parameters $u_1, \cdots, u_m$ and a number of dependent variables $x_1, \cdots, x_n$. Typically, the hypothesis is composed of

$$\begin{cases} h_1(u_1, \cdots, u_m, x_1, \cdots, x_n) = 0, \\ \\ h_r(u_1, \cdots, u_m, x_1, \cdots, x_n) = 0, \end{cases} \tag{1}$$

where the h's are polynomials over a ground field K. The conclusion is

$$g(u_1, \cdots, u_m, x_1, \cdots, x_n) = 0 \tag{2}$$

where g is a polynomial over K.

If the geometric statement is not generically true, by computing Gröbner bases with parameters, we can find the conditions, which the parameters should satisfy, such that the conclusion (2) can be deduced from the hypothesis (1).